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To AdS and back again

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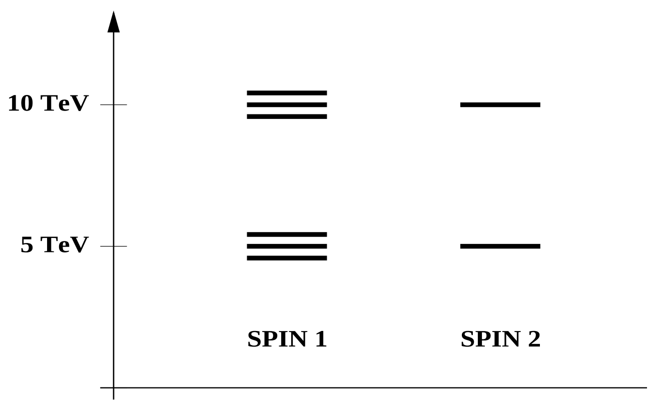
Preface

Let us imagine that the first results of the LHC will describe, at least for the bosons, the spectrum shown at the end of the page.

Any postdoc in theoretical particle physics would be probably convinced that our world is described by the Randall-Sundrum (RS) model [1]: the universe is a 5 dimensional Anti-de Sitter (AdS) space, with a huge cosmological constant. The spin 2 resonances are the first Kaluza-Klein (KK) modes of the graviton. The mass scale $m_W \sim 5$ TeV is the gravitationally red-shifted Planck mass. The spin 1 triplets are the KK modes of the gauge fields which propagate in the AdS bulk: in this case, the $SU(2)_L$ vector bosons of the Standard Model.

At the same time, a physicist who have never cared about extradimensions, looking at the same spectrum, would give a different, purely 4 dimensional, description. What comes to his eyes is nothing but QCD. The massive spin 1 and spin 2 states are the resonances of a new, strongly coupled gauge interaction with a certain number N of colors; the resonances are exactly the same as the ρ and f_2 in cromodynamics. This gauge interaction has a global SU(2) symmetry which is manifest in the degeneracy of the spectrum: the spin 1 states are triplets and the spin 2 are singlets under this symmetry. In the same way, we classify the QCD resonances in terms of isospin representations. And the scale m_W is dynamically generated by dimensional transmutation, like Λ_{QCD}. These two points of view appear very different, but they are both right. The two descriptions in



fact are completely equivalent and the precise “dictionary” between the two is given by the AdS/CFT correspondence [2, 3, 4].

This particular example, and the Randall-Sundrum model in general, could have nothing to do with the real world, but, nevertheless, it has deeply influenced our viewpoint on spacetime. Maybe the hope to answer, looking to the experiments, the question if we live in a universe with 3 or 9 or any other number of spatial dimensions, could be not well-founded. Maybe there is not a “true” number of dimension we should look for, or a “true” spacetime that will be the background for the fundamental theory of nature. Perhaps we could have many descriptions of the same physics, as in our example, in a different number of spatial dimensions. One could be more useful, or simpler, than another in describing a specific physical process, but they are all equivalent. The number of spatial dimension should not be thought as “fundamental”. This is the lesson of the AdS/CFT correspondence and this is the approach I will follow in my Thesis.

The results I present are an extended collection of two papers. In the first, with Roberto Contino and Paolo Creminelli [5], I considered gauge couplings evolution in the Randall-Sundrum model. Then with Thomas Gregoire, Riccardo Rattazzi, Claudio Scrucca and Alessandro Strumia [6] we studied gravitational quantum corrections and supersymmetry breaking in warped brane models. Besides these works, during my PhD, I investigated also the relation between matrix models and supersymmetric gauge theories [7, 8] and the stability of massive gravity [9].

Introduction

The Standard Model, or at least its main structure (symmetries, particle content, charges), was set in the late 70's. For the theoretical particle physics community, the history of the last 25 years, in the absence of striking new data from colliders, has been the search for the right organizing principle that guide it beyond the Standard Model. A generally accepted principle has been the naturalness and the absence of fine-tuning in the fundamental physical scales. *If we accept this principle*[1], we run into two huge problems because we are not able to explain why the cosmological constant Λ and the Higgs mass m_H are incredibly smaller than the cutoff of the Standard Model, the Planck mass M_4. In fact we know from the experiments that $\Lambda \sim 10^{-120}\ M_4^4$ and $m_H^2 \sim 10^{-32}\ M_4^2$.

Up to now, we have almost no idea on how to solve the first problem. The (reasonable) hope and assumption is that, because Λ is associated with gravity, which is not fully understood, the cosmological constant problem will find a separate solution, not affecting physics at the weak scale.

On the contrary, the second one, the gauge hierarchy problem, has been an exemplar source for inspiration. Many ingenious proposals were made in the last 25 years: technicolor, supersymmetric Standard Model, extradimensions and low-scale quantum gravity, little Higgs, all are possible solutions. Every approach to the hierarchy problem introduces new physics at the TeV scale in order to make the electro-weak symmetry breaking (EWSB) natural; but at same time this new physics need not to screw up the SM success in explaining the absence of big flavor violation, CP violation, baryon number violation...

The simplest and more realistic possibility still is low energy supersymmetry. It satisfies almost effortless the constraints posed by electroweak precision data and, among all the theories motivated by naturalness, it is the only one with a concrete, spectacular success: gauge couplings unification. Despite this success, there are still several unsatisfactory aspects, mainly related to the problem of supersymmetry breaking. Insisting on naturalness, one could try to improve these results along two different directions. If we want to stick to supersymmetry, we can concentrate on searching for a simple and realistic theory for the supersymmetry breaking soft terms. Alternatively, since super-

[1] For example, an alternative paradigm, based on the "anthropic" principle [10] and on the existence of an enormous "landscape" of metastable vacua in string theory [11, 12, 13, 14], has led to the proposal of split supersymmetry [15]

symmetry is still considered the favorite approach to hierarchy mainly because of gauge coupling unification (but we could be misreading this hint...), we can ask if there is another proposal that realizes unification with a predictive power at least comparable to that of 4d SUSY GUT.

In this Thesis we investigate both directions using field theory on AdS_5 as a tool to explore new aspects of physics in 4 dimensions. As we saw in the preface for a particular example, 5 dimensional field theory in AdS has a 4d equivalent description along the prescriptions of the AdS/CFT correspondence [2, 3, 4]. The correspondence, and the holographic interpretation of the Randall-Sundrum model, are discussed in chapter 1. This connection between 5 dimensional and 4d quasi-conformal theories could be very useful for gaining new insights on some properties of the strongly-coupled regime, which can be addressed by perturbative calculations in the dual AdS theory.

This approach is used first in chapter 2 to reconsider the problem of supersymmetry breaking. Supersymmetry needs to be broken in order to give weak scale masses to the superpartners. In order for all the superpartners to be heavier than the Standard Model particles, supersymmetry is typically broken in a hidden sector and transmitted to the Standard Model by gravitational [16, 17, 18] or gauge interactions [19]. At low energy, the breaking of supersymmetry is encoded in soft supersymmetry breaking masses. A crucial ingredient in designing a supersymmetry breaking scenario is to ensure that the soft masses do not generate phenomenologically unacceptable flavor changing neutral currents (FCNC). The safest way of doing this is to generate soft masses in the infrared, where the only flavor spurions are the Yuakawa matrices. This insure that there will be a super-GIM mechanism that will suppress FCNC. Gauge mediation is of this type while gravity mediation is not, because the soft masses are generated by divergent gravity loops dominated in the UV. Another supersymmetry breaking transmission mechanism that is safe with respect to flavor is anomaly mediation [20, 21]. In this scenario, supersymmetry breaking is transmitted via the super-Weyl anomaly, and is also dominated by the IR. However, this contribution is parametrically smaller than the gravity mediated contribution. A way to suppress gravity mediated contributions is to invoke an extradimension, and to spatially separate the hidden sector that breaks supersymmetry and the visible sector. By locality, contact interactions between the hidden and the visible sectors are absent at tree level and are generated by finite gravity loops dominated by the IR. In general those loops are subdominant, and the soft masses are entirely generated by anomaly mediation. This scenario is very predictive, but unfortunately predicts tachyonic sleptons. However, in some region of parameter space, the finite gravity mediated contributions can compete with anomaly mediated ones and one could hope that they cure the tachyonic sleptons problem.

In flat space, gravity loops will induce corrections to the Khaler potential of the form:

$$a\frac{\Phi_v^\dagger\Phi_v}{M_5^3\,(T+T^\dagger)^3}+b\frac{\Phi_h^\dagger\Phi_h\Phi_v^\dagger\Phi_v}{M_5^6\,(T+T^\dagger)^4} \tag{1}$$

where Φ_v and Φ_h are respectively the visible and hidden sector superfields, T is the radion superfield whose vev determines the radius, and M_5 is the five dimensional Planck scale. If the radion or Φ_h gets a non-zero F term, the visible fields get soft masses respectively from the first and second terms in (1). The first term is the so-called radion mediated contribution, calculated for the first time in [22] and the second term is the brane to brane mediated contributions and was calculated in [23, 24]. They found that in most cases the soft masses remain negative, but [23] found the possibility of having a positive radion-mediated contribution if a substantial kinetic term for gravity is included on the hidden brane.

The effect of the large kinetic term on the hidden brane is to suppress the wave function of the KK modes at the position of the brane. This is similar to what happen in a warped space, such as in an RS setup [1]. In the second chapter, we compute the full effective Kähler potential in a supersymmetric version of RS. From this result we can compute the gravity mediated and radion mediated supersymmetry breaking in such a space, in the presence of arbitrary kinetic terms on both branes.

The form of the corrections we should expect in warped space can deduced from the AdS/CFT correspondence. In the limit where the Planck brane is sent to $z_0 = 0$, the theory becomes conformal, and the couplings of the radion are dictated by conformal invariance. In this case there are no corrections to the Kähler potential. The presence of the Planck brane explicitly breaks conformal invariance, and corrections to the Kähler potential will be induced from graviton loops attached to the CFT. These loops are cutoff at the compositness scale μ. To estimate the form of these corrections, we just take the tree level kinetic terms, and compute the corrections due to a loop of 4d graviton.

The low energy, tree level kinetic term for a properly defined radion is given by [25]:

$$\frac{M_5^3}{k^3}\int d^4x d^4\theta \mu^\dagger \mu \tag{2}$$

The correction to this term coming from a graviton loop is down by a factor of $\mu^\dagger\mu/M_4^2$:

$$\frac{M_5^3}{k^3}\int d^4x d^4\theta \frac{1}{M_4^2}\left(\mu^\dagger\mu\right)^2 = \frac{1}{k^2}\int d^4x d^4\theta \left(\mu^\dagger\mu\right)^2 \tag{3}$$

In the presence of matter fields Φ_0 on the Planck brane, powers of $\Phi_0^\dagger\Phi_0$ can be attached to a graviton line, giving corrections to the Φ_0 kinetic term of the form:

$$\frac{M_5^3}{k^3}\int d^4x d^4\theta \Phi_0^\dagger\Phi_0 \frac{\left(\mu^\dagger\mu\right)^2}{M_4^4} = \int d^4\theta \frac{1}{k^2}\frac{1}{M_4^2}\Phi_0^\dagger\Phi_0\left(\mu^\dagger\mu\right)^2 \tag{4}$$

A field Φ_1 on the TeV brane with tree level kinetic term

$$\frac{1}{k^2}\int d^4x d^4\theta \mu^\dagger\mu\Phi_1^\dagger\Phi_1 \tag{5}$$

corresponds to a bound state of the CFT. It's kinetic terms, also receive corrections from a loop of graviton:

$$\int d^4x d^4\theta \frac{1}{k^2}\frac{1}{M_4^2}\left(\mu^\dagger\mu\right)^2 \Phi_1^\dagger\Phi_1 \tag{6}$$

We will show that we indeed find corrections of this type in the limit of large warping. When the warping gets smaller, higher powers of $\mu^\dagger\mu/k^2$ become important. Those higher powers can be understood as coming from cutoff effects due to the Planck brane.

From this holographic pictures, it is clear what happens when we add a kinetic term for the graviton on the Planck brane. It just makes M_4 bigger and the corrections only get smaller.

The lessons from these holographic considerations is that, in absence of localized kinetic terms, for large warping, the gravity and radion mediated soft masses go to zero, while for zero warping they are negative. We could hope that, keeping z_1 fixed, when $z_0 \sim 1/k$ one of the radion mediated contribution becomes positive, but we will find that this is not the case. Adding a kinetic term on the Planck brane doesn't help in the case of large warping, but we could hope that it does for intermediate warping. We can also put a kinetic term on the IR brane. However, such a kinetic term cannot be too large, otherwise it leads to ghosts, either for the radion, or for the graviton.

There are two possible techniques that can be used to perform the calculation. One is the use of the component formalism of Zucker [26, 27, 28] as was done for the flat case in [23]. This approach is somewhat inconvenient in the warped case for two reasons. The formalism is plagued with singular products of delta functions that are hard to deal with in warped space. Also, the trick used by [23] of turning on a non-zero F term for the radion and of calculating the potential instead of a Kähler potential, is also hard to generalize to the warped case. Therefore, in this paper, we use the superfield techniques employed in [24]. We review this technique in section 2.3 and generalize it to warped space. We then present the supergraph calculation and show that the non-trivial information of the result can be in fact computed in a simple theory of a scalar field in warped space with localized kinetic terms. This is similar to the computation of the Coleman-Weinberg potential which depends only on the spectrum of the theory in some background. The nature of the fields (gauge field, fermion, scalar) that contribute to the potential is only reflected in an overall constant counting the number of real propagating degrees of freedom.

In chapter 3 we consider GUT models where the gauge bosons propagate in the *non supersymmetric* AdS bulk. The aim of the chapter is to study the evolution of gauge couplings in this setup. We want to compare it with supersymmetric unification, in order to better understand if this result can be reliably considered as a hint in favor of low energy supersymmetry.

As we said, the RS model with gauge bosons in the bulk is dual to a 4d theory with a conformal sector (see chapter 1). This duality allows us to infer that gauge couplings run logarithmically until very high energy, even if new GUT physics, namely the Kaluza-Klein resonances of the unified gauge bosons, appears at the TeV scale revealing the unified character of the fundamental forces [29]. Changing the GUT group and its breaking modifies the properties of the CFT, and consequently the evolution of gauge couplings until unification, not only some minor threshold corrections like in the standard 4d picture.

In this chapter, we study scalar QED with the computation of the gauge field zero-mode propagator in 5d, for different choices of boundary conditions and for a generic scalar bulk mass. In doing that, we choose dimensional regularization that, we believe, is the most economical and transparent regulator which preserves the symmetries of the AdS background.

From the AdS/CFT correspondence we know that the leading CFT running is described by the *tree-level* AdS propagator of the gauge bosons and it is common to any gauge group. Since the CFT beta-function turns out to be positive, we obtain strong limits on the number of colors N if we impose that the gauge couplings remain perturbative up to a standard GUT scale ($\sim 10^{16}$ GeV). Roughly speaking, the CFT has not to be dominant with respect to the other contributions, so that large values of N are forbidden. This in turn implies that subleading CFT contribution is typically negligible. We compute the *one-loop* correction to the gauge propagator in AdS and we interpret it, in the 4d dual theory, as the sum of two contributions: CFT insertion subleading in a $1/N$ expansion and loops of the additional elementary particles coupled to the CFT. These calculations allow us to study different GUT scenarios where the gauge symmetry is broken either by a Higgs mechanism, or by the boundary conditions. We end the chapter with some general conclusions that can be drawn for model building.

CHAPTER 1

The Randall-Sundrum model

We start this chapter discussing some of the main features of a quantum field theory in a slice of AdS_5 and then we describe how the same physics is reproduced by a 4 dimensional "holographic" theory. The first model of this kind was proposed by Randall and Sundrum [1]; the motivation was to explain the big hierarchy between the Planck scale and the electroweak scale as a consequence of the gravitational redshift in extradimensions [1].

The RS model consists in a 5d gravity theory with a bulk cosmological constant Λ_5 and the fifth dimension compactified on a S^1/Z_2 orbifold of radius R. The action, involving also fields and interactions localized at the two boundaries in $y=0$, πR, is:

$$\int d^4x \int_0^{2\pi R} dy \big\{ \sqrt{g}[\frac{1}{2}M_5^3 \mathscr{R}(g) - \Lambda_5] + \delta(y)\sqrt{g_0}[\mathscr{L}_0 - \Lambda_0] + \delta(y-\pi R)\sqrt{g_\pi}[\mathscr{L}_\pi - \Lambda_\pi] \big\} \tag{1.1}$$

where $g_{0,\pi}$ are the induced metrics and Λ_0, Λ_π are the boundary tensions. By $\mathscr{L}_{0,\pi}$ we indicate any interaction involving fields localized at the boundaries. We can find a static solution of the resulting Einstein equations with 4d Poincaré symmetry

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{1.2}$$

provided the following relations hold:

$$\Lambda_5 = -6M_5^3 k^2, \qquad \Lambda_0 = -\Lambda_\pi = -\Lambda_5/k \tag{1.3}$$

In order for our effective field theory to be a valid description, the AdS curvature k should be smaller than the 5d Planck mass M_5. Notice that the 4d geometry is flat at every point y of the internal dimension, but the conformal scale of the metric varies exponentially with it. The massless spin 2 fluctuations around the background (1.2) describe a 4d graviton whose wave function is peaked on the $y=0$ fixed point and whose interactions are set by the 4d Planck mass

$$M_4^2 = M_5^3 \int_0^{2\pi R} dy\, e^{-2k|y|} = \frac{M_5^3}{k}\Big(1 - e^{-2\pi kR}\Big). \tag{1.4}$$

[1] For a detailed review of the Randall-Sundrum model see [30]

The metric (1.2) can be used to redshift 4-dimensional mass parameters. If we place at 0 and πR two copies of a 4d QFT, any direct experimental comparison of the masses of the equivalent states at each brane gives

$$\frac{m_\pi}{m_0} = e^{-k\pi R}. \tag{1.5}$$

If the Standard Model fields live on the π brane, one can explain the huge hierarchy between the electroweak and the Planck scale with a radius R only about 10 times larger than the curvature radius $1/k$. Because of the relative shift of mass scales, the 0 and πR fixed points are called respectively the Planck and the TeV brane. This is a solution to the hierarchy problem if we can stabilize the radius R, which is the VEV of the massless scalar excitation of the metric (1.2), the radion, to its expected value. The stabilization mechanism can be obtained at the classical level [31] or from quantum corrections [32]. This stabilization gets rid of one of the two fine tunings in (1.3); the remaining one is needed to cancel the 4d cosmological constant. This tuning is common to all the other solutions to the gauge hierarchy problem, like supersymmetry or technicolor.

The KK spectrum of the model, consisting in massive spin 2 excitations, is quantized in units of $\mu \equiv ke^{-\pi kR} \sim$ TeV.

1.1 Randall-Sundrum and holography

In order to discuss the 4 dimensional theory equivalent to the Randall-Sundrum model it is useful to rewrite the AdS metric in “conformal” coordinates, making the change of variable $z = e^{ky}/k$:

$$ds^2 = \frac{1}{kz}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right). \tag{1.6}$$

In these coordinates there is no exponential factor, but the locations of the Planck and the TeV brane, respectively z_0 and z_1, are very far apart

$$z_0 = \frac{1}{k} << \frac{1}{k}e^{\pi kR} = z_1. \tag{1.7}$$

The holographic equivalence was initially stated between a theory of gravity in AdS$_{d+1}$ and a field theory in d dimension living on the boundary of the Anti-de Sitter space [2, 3, 4]. The gravity side can be a string theory or a field theory, in the latter case the equivalence is valid up to energies comparable with the cutoff Λ. The boundary of AdS$_{d+1}$ is topologically equivalent to a d dimensional Minkowski space-time with the point at infinity added. In particular, the symmetry group SO$(d,2)$ of AdS$_{d+1}$ acts as the conformal group on the boundary, so that the holographic theory must be a *conformal* field theory (CFT). This AdS/CFT correspondence states that for any field ϕ which propagates in AdS$_{d+1}$ with a certain boundary value ϕ_0, the conformal sector is deformed by a composite operator O whose source is ϕ_0. The dimension of the operator O is fixed by the value of the mass M of the field ϕ. Irrelevant, marginal and relevant perturbations of the CFT correspond to massive, massless and “tachyonic” modes in the gravity theory. The “tachyonic” modes have negative mass squared but, as shown in [33], they do not lead to any instability.

More explicitly: the generating functional $W[\phi_0]$ of the connected CFT correlation functions coincides with the full action of the AdS$_{d+1}$ theory, $\Gamma[\phi_0]$, computed as a function of the boundary field ϕ_0 [3, 4]

$$\langle e^{-\int d^d x O(x)\phi_0(x)}\rangle_{CFT} = e^{-\Gamma[\phi_0]}. \tag{1.8}$$

When the gravity side is described by a perturbative effective field theory, the conformal sector is in a *strongly interacting* regime, with a *large number N of colors*. Therefore we can use this correspondence to compute non-perturbative quantities of the CFT in terms of perturbative ones in the gravity theory.

To be precise, both sides of eq. (1.8) need to be regularized [4]: the gravity action suffers an infrared divergence, even if computed at the classical level, due to the infinite volume of AdS. On the other hand, though a pure conformal field theory has no divergences, this is not true for its deformation by a composite operator O: correlator functions contain UV divergences at coincident points. In order to make sense of the equation (1.8) we introduce an IR regulator in the AdS theory. The standard procedure is to limit the space-time integration to the region $z > \varepsilon$, this is equivalent to putting a brane at $z = \varepsilon$ (what we previously called the Planck brane), add a proper local counterterm action (divergent for $\varepsilon \to 0$), function of the boundary value of the fields, and then take the limit $\varepsilon \to 0$. This procedure corresponds to some UV regularization of the CFT. At the end we obtain an operative definition of (1.8):

$$\langle e^{-\int d^4x\, O(x)\phi^0(x)}\rangle_{\text{CFT}} = \lim_{\varepsilon\to 0} e^{-\Gamma(\phi_0,\varepsilon)} e^{-\Gamma_{\text{count}}(\phi_0,\varepsilon)} . \tag{1.9}$$

Suppose now not to perform the final limit, keeping an explicitly truncated AdS, and integrate over the boundary values ϕ_0: we obtain the RS model with one brane [34]. Integrating over the boundary, the source ϕ_0 becomes dynamical and describes an elementary field ϕ coupled to the CFT through the interaction $O\phi$. This happens for any field with Neumann boundary condition on the Planck brane, and in particular for the graviton.

Summarizing, the RS theory with localized gravity in an infinite fifth dimension has an equivalent description in terms of a theory in 4 dimensions with a strongly coupled, conformal sector weakly gauged by gravity [35, 36, 37, 38]. If ϕ has Dirichlet boundary condition on the Planck brane, the source is forced to vanish at $z = \varepsilon$ and no elementary field appears in the holographic theory. The Planck brane breaks the translational invariance in AdS and this is translated into an explicit breaking of the conformal invariance at high energies due to the UV cutoff. This breaking is peculiar because no relevant or marginal deformation are introduced in the CFT. Indeed, the only source of conformal breaking in the CFT sector is represented by M_4-suppressed composite operators which are irrelevant at low energy.

On the other hand, adding a TeV brane in the 5d theory corresponds to a *spontaneous* breaking of conformality at low energy [39, 40]. The massless Goldstone field is the radius of the extradimension, before the stabilization. In fact, when a stabilization mechanism is introduced, the radion acquires a mass, indicating that a source of explicit breaking of conformal invariance has been turned on. For instance, the Goldberger-Wise [31] mechanism has been shown to be equivalent to the introduction of a quasi-marginal deformation of the CFT [39, 40]. Because of the conformal breaking at low energy, bound states are generated in the CFT sector, with difference in mass of order $\sim$ TeV. One expects that, if Λ ($\Lambda_T = \Lambda z_0/z_1$) is the cutoff for an observer on the Planck (TeV) brane, there will be $\sim z_1\Lambda_T$ bound state with masses lighter than Λ_T. Their lifetime is sufficiently long that they can be thought of as particles, they are narrow bound states. The remaining modes with masses heavier than Λ_T correspond to broad resonances of the strongly interacting CFT: their lifetime is so short that they form a continuum and they cannot be considered as particles anymore. In practice, the coupling $O\phi$ implies that the CFT bound states mix with the elementary modes already at tree-level [39, 32]. By holographic equivalence, the physical spectrum of the resonances must coincide with the Kaluza-Klein level of the 5d theory.

Also brane localized fields have a holographic counterpart [39, 40]. Fields on the Planck brane correspond, in the 4d theory, to elementary degrees of freedom external to the CFT, while fields on the TeV brane are interpreted as bound states of the conformal sector, which enter a strong coupling regime at energies greater than the TeV.

In the original Randall-Sundrum model only gravity propagates in the bulk of the warped dimension, while the Standard Model sector is confined on the TeV brane. The holographic equivalent of this setup is a 4d theory of gravity where the Standard Model is embedded in a strongly interacting conformal sector with a large number of colors. In the next section we describe which is the theory dual to RS when also gauge bosons propagate in the AdS bulk.

1.2 Holographic gauge bosons

A *gauge* invariance G in pure AdS is equivalent to a CFT with a *global* symmetry G [4]. In particular, the composite operator associated to a bulk gauge field A_μ is the conserved Noether current J_μ. When the two branes are introduced, if the bulk gauge symmetry is not reduced by boundary conditions, the 4d theory is a CFT with the global invariance weakly gauged by an external vector. Suppose now that the bulk group G is reduced to a subgroup H_0 on the Planck brane: only the components of the gauge field A_μ which do not vanish on the brane will be associated to elementary vectors in the holographic theory. Only the subgroup H_0 of the global CFT symmetry is gauged. On the other hand, when the symmetry is reduced to a subgroup H_1 on the TeV brane, in the dual theory the strong dynamics of the CFT induces a spontaneous breaking $G \to H_1$ of its global invariance.

Following [41], let us consider the most general case of a bulk gauge group G reduced to the subgroups H_0, H_1 on the Planck and TeV branes respectively. Then the parity assignment for the gauge fields will be:

$$\begin{array}{ll} A^a_\mu\ (+,+) & T^a \in \mathsf{Alg}\{H\}, \\ A^{\bar{a}}_\mu\ (+,-) & T^{\bar{a}} \in \mathsf{Alg}\{H_0/H\}, \\ A^{\tilde{a}}_\mu\ (-,+) & T^{\tilde{a}} \in \mathsf{Alg}\{H_1/H\}, \\ A^{\hat{a}}_\mu\ (-,-) & \end{array} \tag{1.10}$$

where $H = H_0 \cap H_1$, by $+$ $(-)$ we denote the Neumann (Dirichlet) boundary condition. The A_5's have the opposite boundary conditions to those of the corresponding A_μ's.

The 4d theory contains a CFT sector with a global invariance G spontaneously broken down to H_1 by the strong dynamics. The external gauge fields weakly gauge only the subgroup H_0 of G:

$$\mathscr{L} = \mathscr{L}_{\mathrm{CFT}} - \frac{1}{4g^2}(F^\alpha_{\mu\nu})^2 + A^\alpha_\mu J^{\mu\,\alpha}, \qquad \alpha = a, \bar{a}. \tag{1.11}$$

The gauging of only a subgroup of its global symmetry G is experienced by the CFT as an explicit breaking. Let us count the Goldstone bosons: some are eaten by the $A^{\hat{a}}_\mu$ gauge fields to form massive vectors, but n of them, n = dim(G/H_1) - dim(H_0/H), are "holographic" pseudo-Goldstones. They are massive at tree level, because the explicit breaking comes only from the interaction $J \cdot A$, but they are expected to acquire mass from radiative corrections. This is very similar to the pion mass splitting in QCD with massless quarks. The electromagnetic interaction breaks explicitly the global chiral symmetry and the charged pion gets mass at 1-loop.

The 4d theory and the RS describe the same physics, so they must have the same spectrum. The n massless scalars, with the same quantum number of the pseudo-Goldstone, are the zero modes of the $A^{\hat{a}}_5(+,+)$ in the tree-level Kaluza-Klein spectrum. Their wave function is exponentially localized toward the TeV brane, suggesting that their holographic counterparts are bound states of the CFT.

In general, any interaction with the elementary sector, for example with a fermion, and not only the gauging (1.11), will communicate to the CFT the explicit breaking of G. In fact an elementary field ϕ will come in a representation of the group that survives on the Planck brane, H_0, rather than G. The coupling ϕO_ϕ with the CFT is not G-invariant and a mass term for the pseudo-Goldstone boson is thus generated only in processes where an elementary field is exchanged.

1.3 Supersymmetric Randall-Sundrum

In the second chapter, we show how brane world scenarios are a geometric realization of the so-called hidden sector supergravity models and why they could help in finding a realistic theory for the supersymmetry breaking soft terms. For this reason we need a supersymmetric version of the Randall-Sundrum model. Since we require an $N=1$ supersymmetric four dimensional effective theory and because the Z_2 projection eliminates half of the supersymmetry, we have to start with an $N=2$ supergravity theory in the bulk. The bulk has to contain a cosmological constant, which is consistent with supersymmetry only if the vacuum is an anti-de Sitter space. An AdS supergravity needs a gauging of the $\mathscr{R}$-symmetry. The minimal choice is to gauge a U(1) subgroup by the graviphoton, since no additional fields are introduced.

There are at least three methods to construct the lagrangian. Two are on-shell, the basic difference is in the parity of the gauge coupling constant. In the first it is taken to be odd [42, 43] while in [44] it is even. The third approach is off-shell and was developed by Zucker [28]. We follow the first method. Using the same notation of the previous sections we have:

$$\mathscr{L} = \mathscr{L}_5 + \delta_0(y)\mathscr{L}_0 + \delta_1(y-\pi R)\mathscr{L}_1 \,, \tag{1.12}$$

where

$$\begin{aligned}
\mathscr{L}_5 &= \sqrt{g_5}\Big[-\Lambda_5 - \frac{1}{2}M_5^3\Big(\mathscr{R}_5 + i\bar{\Psi}_M(\Gamma^{MRN}D_R - \tfrac{3i}{2}k\varepsilon(y)\Gamma^{MN})\Psi_N + \tfrac{1}{2}F_{MN}^2\Big) + \cdots\Big] \,,\\
\mathscr{L}_i &= \sqrt{g_4}\Big[-\Lambda_i - \frac{1}{2}M_i^2\Big(\mathscr{R}_4 + i\bar{\Psi}_\mu\gamma^{\mu\rho\nu}D_\rho\Psi_\nu\Big) + \Big(|\partial_\mu\phi_i|^2 + i\bar{\psi}_i\gamma^\mu D_\mu\psi_i\Big) + \cdots\Big] \,.
\end{aligned}$$

The bulk and boundary cosmological constants are tuned as usual (1.3) while $M_{0,1}$ are two scales parameterizing possible localized kinetic terms for the bulk fields. The above theory has a non-trivial supersymmetric warped solution defining a slice of an AdS_5 space that is delimited by the two branes at $y=0,\ \pi R$. The background is given by

$$g_{MN} = e^{-2k|y|}\eta_{\mu\nu}\delta_M^\mu\delta_N^\nu + \delta_M^y\delta_N^y \,, \qquad \Psi_M = 0 \,, \qquad A_M = 0 \,. \tag{1.13}$$

CHAPTER 2

Supersymmetry breaking

Low energy supersymmetry is, up to now, the simplest and the more realistic possibility for a solution to the hierarchy problem, with new physics at the electroweak scale. It predicts gauge couplings unification, electroweak symmetry breaking is triggered radiatively by the top Yukawa interaction, the little hierarchy is satisfied and there is a natural candidate for cold dark matter. However, superparticles have not been detected yet and no deviations from the Standard Model have been seen in precision measurements. This means that not only SUSY must be broken to give sparticles a mass, but also that the mechanism for SUSY breaking should be clever enough if it has to describe the observed features of the low energy world.

In the next paragraph we briefly enumerate the desired features of a theory of SUSY breaking and then we focus on the supersymmetric flavor problem.

The first requirement of any successful SUSY breaking scenario is to give correct masses to the superpartners. At the present days, after the completion of the LEP program without discovering any superparticle, we know that sparticle masses are somewhat heavier than expected. The present lower bound on sparticle masses requires a fine tuning of at least 1/20 among the parameters of all popular models. Is this fine tuning accidental or is there a model which naturally selects a heavier spectrum?

The second, strong, experimental constraint on the structure of the squark and slepton mass matrices comes from the Flavor Changing Neutral Currents (FCNC). In the Standard Model all flavor violation comes from the fermion mass matrices. FCNC are naturally suppressed, in agreement with experimental data, by the GIM mechanism. In the Minimal Supersymmetric Standard Model (MSSM) a generic sfermion mass matrix represents a new source of flavor mixing. Why the ultraviolet squark mass matrix and running effects should be such as to produce the extreme level of degeneracy demanded phenomenologically is the supersymmetric flavor problem.

Third, the parameter multiplying $H_u H_d$ in the superpotential, the μ parameter, should be between about 100 GeV and 1 TeV. It does not itself break supersymmetry but one needs to connect its magnitude to that of SUSY breaking soft terms in a simple way. This can be an obstacle in the construction of a realistic theory.

Last, the phases of the A and B parameters are constrained to be small. That is, CP should be approximately conserved.

2.1 The susy flavor problem and extradimensions

In this chapter we will consider scenarios in which SUSY breaking is communicated to the visible sector by (super)gravity. We are interested in deriving the consequences of the presence of extradimensions on the sfermion mass matrix and on the supersymmetric flavor problem. For this purpose it is enough to consider a visible sector, the toy MSSM, consisting of just one chiral superfield Φ_0, containing a sfermion ϕ_0, a Weyl fermion χ_0 and the auxiliary field F_{ϕ_0}. We don't know the details of the presumably strongly coupled, supersymmetry breaking sector but all we need is to assume that it can be effectively described again by another single chiral superfield, Φ_1. We also assume that all the interactions in the hidden sector are characterized by just one scale Λ_H. The superfield Φ_1, whose auxiliary component has a VEV $\sim \Lambda_H$, can be interpreted as the effective low energy description of a dynamical SUSY breaking model.

Let's start considering the standard 4 dimensional gravity mediated model [16, 17, 18]. We find very convenient using the superconformal formulation of SUGRA [45, 46, 47]. In this formulation, one first constructs an action invariant under a bigger gauge group, the local superconformal transformations; then breaks local superconformal symmetry explicitly down to local super-Poincaré to define the lagrangian. Every field is assigned a Weyl weight (scaling dimension) and the lagrangian is made conformal invariant introducing a *compensator* field. Conformal invariance is then broken fixing the extra gauge symmetries. We choose a chiral compensator S with Weyl weight +1 and, after the superconformal gauge fixing, we have

$$S \equiv 1 + F_S \Theta^2 \tag{2.1}$$

where F_S is the scalar auxiliary field of the off-shell $\mathcal{N} = 1$ supergravity multiplet. The superfield S will enter in a way precisely determined by the conformal scaling of any operator in the lagrangian. The general action for chiral matter in a supergravity theory is then:

$$\mathcal{L} = \left[\Omega(\Phi_{0,1}^\dagger, \Phi_{0,1}) S^\dagger S\right]_D + \left[P(\Phi_{0,1}) S^3\right]_F + \left[P(\Phi_{0,1}) S^3\right]_F^\dagger . \tag{2.2}$$

The functions Ω and P have an expansion of the type

$$\Omega = -3M_4^2 + \Phi_0^\dagger \Phi_0 + \Phi_1^\dagger \Phi_1 + \frac{c}{M_4^2} \Phi_0^\dagger \Phi_0 \Phi_1^\dagger \Phi_1 + \ldots \tag{2.3}$$

$$P = \Lambda^3 + \Lambda_H^2 \Phi_1 + \ldots \tag{2.4}$$

The parameter Λ in the superpotential, as usual, is tuned to cancel the cosmological constant; c is an order one dimensionless coupling. Ω can be written as a function of the Kähler potential K

$$\Omega \equiv -3M_4^2 e^{-K/3M_4^2} . \tag{2.5}$$

The supergravity lagrangian (2.2) is not in the canonical form where the Einstein action has the field-independent coefficient $M_4^2/2$. To obtain the canonical form one has to redefine the metric by a Weyl transformation.

Now we will focus on the *tree-level* contribution to the scalar mass squared that comes from the lagrangian (2.2). When supersymmetry is broken, two things carry this information: the VEV of the auxiliary field in the hidden sector multiplet and in the compensator multiplet. The second does not contribute to the tree-level ϕ_0 mass because we can eliminate the S dependence by rescaling Φ_0 according to

$$\Phi_0 S \to \Phi_0. \tag{2.6}$$

So the *only* source of tree-level scalar mass squared is the *direct* coupling between the hidden and the visible sector in (2.3)[1]. After putting in the hidden sector VEV, we find the soft masses

$$m_{\phi_0}^2 = c\frac{\Lambda_H^4}{M_4^2}. \tag{2.7}$$

The *flavor problem* resides in the fact that there is no reason for the c couplings of the sfermions to the hidden sector, and consequently the ultraviolet sfermion mass matrices, to respect flavor. The source of this non-renormalizable coupling is that we have integrated out the Planck-scale states, which may couple to both the visible and hidden sectors. The unknown fundamental theory, that we have integrated out, has to explain why the top quark is much heavier than the up quark and everything else: it should be also the theory of flavor. Then there is absolutely no reason to believe that the couplings are flavor-blind.

What does it change if we start from an initially five dimensional theory? Suppose that the theory is compactified on S^1/Z_2 and that the visible sector is confined to a 3-brane localized at the fixed point $z = z_0$ (z is the coordinate of the fifth dimension) while the hidden sector is at $z = z_1$. This kind of scenario is called ”sequestered” [20].

The lagrangian (2.2) is completely general and can be also the effective 4d description of a higher dimensional theory, below the compactification scale μ_1. Since the radius is also a massless field, we will have to include it in the effective 4d description and to determine the vacuum dynamics; we call T the radion chiral superfield. As a consequence, the functions Ω and P will depend not only on Φ_0 and Φ_1 but also on T. The essential point, coming from the higher dimensional origin, is that there is *no direct coupling*, at tree-level, between the visible and the hidden sector, because they are truly separated in the fifth dimension [20]. It follows that Ω and P must take the special form:

$$\Omega_{cl} = -3M_4^2(T+T^\dagger) + \Omega_0(\Phi_0, \Phi_0^\dagger, T+T^\dagger) + \Omega_1(\Phi_1, \Phi_1^\dagger, T+T^\dagger) \tag{2.8}$$

$$P_{cl} = P_0(\Phi_0, T) + P_1(\Phi_1, T) \tag{2.9}$$

[1] This is not true at the quantum level because the rescaling (2.6) is anomalous as we will discuss in section 2.2.1

the function $M_4^2 = M_4^2(T+T^\dagger)$ becomes the 4d Planck mass when we replace the radion by its vacuum expectation value; Ω_0 and Ω_1 are the contributions to the gravitational kinetic function coming respectively from the z_0 and z_1 fixed points. In the simplest situation we have:

$$\Omega_i(\Phi_i,\Phi_i^\dagger) = \Phi_i\Phi_i^\dagger + \dots \qquad i=0,1 \tag{2.10}$$

the dots denote irrelevant operators involving higher powers of $\Phi_i\Phi_i^\dagger$. The dependence on $T+T^\dagger$ is fixed by the geometry of the fifth dimension: for a flat extradimension we have

$$\Omega_{cl} = -\frac{3}{2}(T+T^\dagger)M_5^3 + \Omega_0(\Phi_0,\Phi_0^\dagger) + \Omega_1(\Phi_1,\Phi_1^\dagger) \tag{2.11}$$

$$P_{cl} = P_0(\Phi_0) + P_1(\Phi_1) \tag{2.12}$$

while for a warped extradimension, with curvature radius $1/k$,

$$\Omega_{cl} = -\frac{3}{2}\frac{M_5^3}{k}(1-e^{-k(T+T^\dagger)}) + \Omega_0(\Phi_0,\Phi_0^\dagger) + \Omega_1(\Phi_1,\Phi_1^\dagger)\, e^{-k(T+T^\dagger)} \tag{2.13}$$

$$P_{cl} = P_0(\Phi_0) + P_1(\Phi_1)\, e^{-kT}. \tag{2.14}$$

The absence of the $1/M_4^2$ direct coupling between the two sectors has the potential to eliminate dangerous flavor violation. Of course, in principle a four-dimensional theory could have a Kähler potential of the special form (2.8); however, there is no symmetry which would maintain it in the presence of radiative corrections. In a theory with extradimensions this separation is ”natural” in the sense that it is enforced by geometry. On the other hand, it is consistent to assume that, even in 4 dimensions, there is no interaction in the superpotential because the relation (2.9) is radiatively stable due to the non-renormalization theorem.

Although the visible and hidden sectors are decoupled at tree-level, there will be couplings generated radiatively between the two sectors. Nevertheless, because there is *no counterterm involving hidden and visible sector fields*, these couplings arise from a *finite* supergravity calculation. Moreover, the introduction of a *new scale* μ_1, assumed to be parametrically smaller than the 5d Planck scale M_5, implies that higher loops are further suppressed by powers of μ_1/M_5.

2.2 Soft scalar masses

In the ”sequestered” model there is another potential source for the soft masses, F_T, besides the two already present in the 4d theory. However, it is easy to see that scalar masses are *not generated classically*. In fact S can be again redefined away by $\Phi_0 S \to \Phi_0$, and both F_T and F_{Φ_1} do not couple directly to the visible sector[2], as we can see from (2.11)

[2]Even if we are not discussing gaugino masses, notice that S does not couple to the gauge fi eld strength, because $\mathscr{W}^2$ is scale invariant, and then also the gaugino mass vanishes at tree-level.

and (2.13). Soft masses will indeed be generated at the radiative level, and the leading contribution to the sfermion mass matrix will be *flavor symmetric*. In the following two subsections we will describe the *two calculable quantum effects* that give the leading contributions to the sfermion masses: the so called anomaly mediated supersymmetry breaking (AMSB) [20, 21] and the one-loop correction $\Delta\Omega$ to Ω_{cl}, which introduces direct couplings between visible, hidden and radion sectors.

2.2.1 Anomaly mediated SUSY breaking

Although we have showed that there are no tree-level supersymmetry breaking masses, this is not true at the quantum level. The rescaling $\Phi_0 S \to \Phi_0$ is anomalous and it generates a two loop scalar mass squared.

For the purpose of discussing the anomaly mediated contribution we introduce in the visible sector also a vector multiplet V, which actually is present in the MSSM. We integrate out the hidden sector and the only contribution of its dynamics is through the field S (that is gravity) which couples to both sectors. The effective lagrangian is

$$\mathscr{L}_{\text{eff}} = \left[\Phi_0^\dagger e^{-V}\Phi_0 S^\dagger S\right]_D + \left[S^3(m_0\Phi_0^2 + y_0\Phi_0^3) + \frac{1}{g_0^2}\mathscr{W}_\alpha^2\right]_F + \text{h.c.} + \mathscr{O}(1/M_5). \quad (2.15)$$

The field $< S >= 1 + F_S\Theta^2$ appears as a supersymmetry breaking background. We see that in this effect gravity enters only at the classical level. We can eliminate the S dependence rescaling Φ_0 as before:

$$\Phi_0 S \to \Phi_0 \quad (2.16)$$

The naive result is then

$$\mathscr{L}_{\text{eff}} = \left[\Phi_0^\dagger e^{-V}\Phi_0\right]_D + \left[m_0 S\Phi_0^2 + y_0\Phi_0^3 + \frac{1}{g_0^2}\mathscr{W}_\alpha^2\right]_F + \text{h.c.} \quad (2.17)$$

As we expect, S appears only in front of the visible mass operator. Let's assume that there are no explicit mass parameters in the lagrangian, $m_0 = 0$. Classically the S dependence, and hence supersymmetry breaking, are again absent, however the quantum functional integral measure is not invariant under the rescaling (2.6). Now we evaluate the anomalous S dependence, and the scalar mass squared, simply by using two facts:

1. although there are no explicit mass terms, the ultraviolet cutoff always provides an implicit mass scale for the theory, then S will multiply cutoff dependence;

2. the lagrangian (2.15) has an exact formal R-symmetry, under which, before the rescaling, R[S]=2/3, R[Φ_0]=0 . This symmetry is valid even in the presence of a mass term, so it is exact even when we introduce an ultraviolet regulator field.

Let's renormalize the theory (2.17) at an infrared scale μ. The effective lagrangian must take the general form

$$\mathscr{L}_{\text{eff}} = \left[\mathscr{Z}\left(\frac{\mu}{\Lambda_{UV}S}, \frac{\mu}{\Lambda_{UV}S^\dagger}\right)\Phi_0^\dagger e^{-V}\Phi_0\right]_D + \left[y_0\Phi_0^3 + \tau\left(\frac{\mu}{\Lambda_{UV}S}\right)\mathscr{W}_\alpha^2\right]_F + \text{h.c.} \quad (2.18)$$

The dependence on μ/Λ_{UV} follows from dimensional analysis and the presence of S is a consequence of the point 1. The non-renormalization theorem for the superpotential implies that the cutoff Λ_{UV} appears only in the Kähler potential and in the gauge coupling (which renormalizes only at one-loop). τ must be a holomorphic function and can depend only on S, while the superfield $\mathscr{Z}$ must be a function of both S and $S^\dagger$.

Because of the second point, $\mathscr{Z}$ must depend only on the R-invariant combination

$$|S| \equiv (S^\dagger S)^{1/2}; \tag{2.19}$$

then from now on we will consider the superfield $\mathscr{Z}$ to be a function of $\mu/(\Lambda_{UV}|S|)$.

At this point we can Taylor-expand the $\log S$ dependence in the Kähler potential, recalling that $S = 1 + F_S\Theta^2$:

$$\begin{aligned}\log\mathscr{Z}(\frac{\mu}{\Lambda_{UV}|S|}) &= \log Z(\frac{\mu}{\Lambda_{UV}}) - \frac{1}{2}F_S\Theta^2\frac{d\log Z}{d\log\mu}(\frac{\mu}{\Lambda_{UV}}) + \text{h.c.}\\ &+\frac{1}{4}|F_S|^2\Theta^2\bar{\Theta}^2\frac{d^2\log Z}{d(\log\mu)^2}(\frac{\mu}{\Lambda_{UV}}) \\ &\equiv \log Z(\frac{\mu}{\Lambda_{UV}}) - \frac{1}{2}\gamma(g,y)(F_S\Theta^2+\text{h.c.}) + \frac{1}{4}\dot{\gamma}(g,y)|F_S|^2\Theta^2\bar{\Theta}^2\end{aligned} \tag{2.20}$$

where we have defined

$$\begin{aligned}\gamma(g,y) &\equiv \frac{d\log Z}{d\log\mu}\\ \dot{\gamma}(g,y) &\equiv \frac{d\gamma}{d\log\mu} = \frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\\ \beta_g(g,y) &\equiv \frac{dg}{d\log\mu}\\ \beta_y(g,y) &\equiv \frac{dy}{d\log\mu}.\end{aligned} \tag{2.21}$$

and $g(\mu)$, $y(\mu)$ are the renormalized couplings. Now we can immediately obtain the scalar mass squared renormalized at μ. In fact, as discussed in [48], the soft terms associated to a chiral superfield Φ_0 can be collected in the running superfield wave function $\mathscr{Z}(\mu)$ in such a way that:

$$\log\mathscr{Z} = \log Z + (A\Theta^2 + \text{h.c.}) - m_0^2\Theta^2\bar{\Theta}^2. \tag{2.22}$$

Comparing eq. (2.20) and (2.22) we find[3]

$$m_0^2(\mu) = -\frac{1}{4}\dot{\gamma}(g,y)|F_S|^2. \tag{2.23}$$

As we have already pointed out, the scalar mass squared arises at two-loops. The masses generated in this scenario are also *flavor universal* because they are functions only of the

[3]With a similar argument, starting from τ, we can also derive the one-loop contribution to the gaugino mass [20, 21]

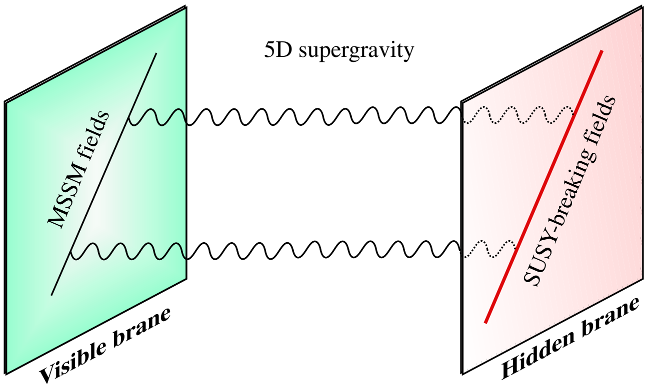


Figure 2.1: *One-loop supergravity diagrams inducing an effective interaction between visible and hidden sector.*

low-energy gauge and Yukawa couplings. From this result we can see the main problem of AMSB: eq. (2.23) can be rewritten as

$$m_0^2(\mu) = \frac{c_0 b_0}{8\pi^2}\alpha^2(\mu)|F_S|^2 + \text{yukawa effects} \tag{2.24}$$

where $c_0 > 0$ is the quadratic Casimir and b_0 is the coefficient of the beta-function. This contribution is therefore positive for asymptotically free gauge theories and negative for infrared free theories. In the MSSM both $SU(2)_L$ and $U(1)_Y$ have $b < 0$. The sleptons masses are determined essentially by the $SU(2)_L \times U(1)_Y$ gauge interactions because Yukawa coupling effects are negligible for at least the first two families. They are therefore *tachyonic*.

We conclude emphasizing that this communication of supersymmetry breaking is present in *any* hidden sector model. In most of them, for example in 4d gravity mediation (2.3), there are larger contributions to the scalar masses at tree-level. However, as we have discussed, this is not true for the sequestered models.

2.2.2 One-loop correction $\Delta\Omega$

In the previous section we have already emphasized that a direct coupling between the visible and the hidden sector is generated at the quantum level through virtual graviton exchange, see fig. 2.1. This loop effect is finite and calculable because locality in 5d insures the absence of counterterms. The loops are saturated at virtuality of the order of the compactification scale μ_1 and, since gravity is flavor universal in the infrared, they induce a universal scalar mass squared. The hope is that this contribution is positive, overcoming the tachyon problem of AMSB. This effect was computed, for a *flat* extradimension, in [23], together with the IR saturated part of the full 1-loop correction to the effective Kähler potential (2.11) at the compactification scale:

$$\Delta\Omega = \frac{\zeta(3)}{4\pi^2(T+T^\dagger)^2} + \frac{\zeta(3)}{6\pi^2}\frac{\Phi_0\Phi_0^\dagger + \Phi_1\Phi_1^\dagger}{(T+T^\dagger)^3 M_5^3} + \frac{\zeta(3)}{6\pi^2}\frac{\Phi_0\Phi_0^\dagger\Phi_1\Phi_1^\dagger}{(T+T^\dagger)^4 M_5^6} + \ldots \tag{2.25}$$

Some comments on this result are in order. Let us start with the third term. It gives the brane-to-brane mediation of SUSY breaking of fig. 2.1. By simple dimensional analysis, we could have guessed the order of the universal scalar mass[4]:

$$m_0^2 \sim \frac{1}{16\pi^2}\frac{|F_{\Phi_1}^2|}{M_5^6(\pi R)^4}. \tag{2.26}$$

This effect becomes negligible in the limit $RM_5 \to \infty$ and, although AMSB scalar masses squared (2.24) arise at two-loop, they dominate eq. (2.26) for $(M_5\pi R)^3 > 16\pi^2 \equiv (M_5\pi R_{cr})^3$ (naive dimensional analysis estimates that quantum gravity effects become important around the energy $\Lambda_5 \sim 4\pi M_5$). Then, in order to compensate the negative contribution of AMSB, the first necessary condition which is required is to *stabilize the radius* at the critical value R_{cr}. Notice that $1/R_{cr}$ is still parametrically smaller than the cutoff. In ref. [49] a simple mechanism of radius stabilization which can plausibly give $R \sim R_{cr}$ was pointed out. Unfortunately, even if the radius is stabilized at the critical value, the brane-to-brane mediation of SUSY breaking in (2.25) has the *wrong sign* and gives again a negative mass squared.

The one-loop correction $\Delta\Omega$ contains another contribution to the scalar masses: the second term in (2.25) induces *radion-mediated* SUSY breaking if the radion field T also gets a non-zero F term. Again it is non-local, because it depends on $(T+T^\dagger)^{-3}$, and calculable. It was computed for the first time in [22]. However, also this contribution turns out to be negative.

In conclusion, in the most minimal case, we have $m_0^2 < 0$ for any R. However in [23] a crucial observation was made. The authors of this paper computed the one-loop correction in a more general situation, where the gravitational multiplet has constant *localized kinetic terms* on the two branes. In this case the minimal quadratic Kähler potentials Ω_i in (2.10) are modified:

$$\Omega_i(\Phi_i, \Phi_i^\dagger) = -3M_i^2 + \Phi_i\Phi_i^\dagger \tag{2.27}$$

The important result is that m_0^2 is usually negative but the *radion-mediated contribution can make it positive* if the kinetic term on the hidden brane is large enough[5]. This is the starting point for the rest of this chapter. A flat extradimension with large localized kinetic terms on one brane is very similar to a warped extradimension, because the effect of the kinetic term is to suppress the wave functions of the KK modes at the position

[4]Gaugino masses are not affected by the brane-to-brane loops

[5]The authors of [23] discuss in detail how anomaly mediation and brane-to-brane effects may cooperate to give a *realistic* sparticle spectrum

of the brane. In the following we compute the 1-loop correction to the Kähler effective potential in the warped case (2.13).

A first part of these corrections represents an uninteresting divergent renormalization of the local operators corresponding to the classical expression (2.13), and are not even reliably computable within an effective theory approach. A second part of these corrections leads instead to new non-local effects that have a radion dependence differing from the one implied by locality in (2.13), and are therefore finite and calculable. These corrections have the following form:

$$\Delta\Omega(T+T^\dagger,\Phi_i,\Phi_i^\dagger)=\sum_{n_0,n_1=1}^{\infty}C_{n_0,n_1}(T+T^\dagger)(\Phi_0\Phi_0^\dagger)^{n_0}(\Phi_1\Phi_1^\dagger)^{n_1}\ . \tag{2.28}$$

The functions C_{n_0,n_1} control the leading effects allowing the transmission of supersymmetry breaking from one sector to the other, and can be computed along the same lines as for the flat case, which was studied in refs. [23] and [24].

Unfortunately the trick that was used in ref. [23], namely computing the effective potential in the very peculiar situation where $F_T \neq 0$ and deducing from it the general form of the above functions before supersymmetry breaking, cannot be generalized in a straightforward way to the warped case. In the flat case a consistent tree level solution with $F_T \neq 0$ and flat 4d (and 5d) geometry could be found by simply turning on constant superpotentials W_0 and W_1 at the boundaries. Since the Kähler potential corresponds to terms quadratic in F_T and since $F_T \propto W_0+W_1$ it was enough to work with infinitesimal $W_{0,1}$ and calculate the effective potential at quadratic order in $W_{0,1}$. Notice, by the way, that boundary superpotentials are just the only local, zero derivative deformation available in 5d Poincare supergravity. The situation is drastically modified in 5d AdS supergravity. In this case by turning on boundary superpotentials we have that [50]: 1) the radius is stabilized, 2) the 4d metric of the 4d slices (and of the low energy effective theory) becomes AdS4, 3) there is a (compact) degeneracy of vacua associated to the vacuum expectation value of the graviphoton A_5. At all points the scale of supersymmetry breaking is subdominant to the scale of AdS4 curvature and at a special point supersymmetry is restored. The last property can be quantified by the deviation $\delta m_{3/2}$ of the gravitino mass from its supersymmetric value $1/L_4$ in AdS4. One obtains $\delta m_{3/2}L_4 \leq \omega^2$ (where ω is the warp factor). In principle one could go ahead and calculate corrections to the effective potential in this background and from it read back the effective Kähler potential. However 4d curvature, as we said, cannot be treated as a subleading effect and this complicates both the calculation and the indirect extraction of the Kähler potential. Rather than trying to encompass this difficulty, we will generalize to the warped case the linearized superfield approach that was used in [24] and in which the Kähler potential is calculated directly. This generalization is interesting on its own, and we will present it in detail in the next section. However, it turns out that the result that it produces for the correction to the effective Kähler potential is a very intuitive and obvious generalization of those derived in [23] and [24] for the flat case: one has just to replace all the Green-functions of the flat Laplacian with those of the warped one. We shall prove this general result in a rigorous way with superfield techniques in the next section, and postpone to yet a subsequent section the actual evaluation of the correction in the general case of a warped space.

2.3 Superspace description

A convenient way of performing loop calculations in supersymmetric theories is to use supergraph techniques. By calculating loop diagrams directly in terms of superfields, the number of graphs is greatly reduced, and various cancellations between graphs that are insured by supersymmetry are guaranteed to happen by the use of superfields. Unfortunately, the use of this technique in theories with more than four space-time dimensions is not straightforward, because the amount of supersymmetry is higher than in four dimensions. From a four-dimensional perspective, there are in this case several supercharges, and the simultaneous realization of the associated symmetries requires a superspace with a more complex structure. However, it is still possible to manifestly realize one of the four-dimensional supersymmetries on a standard superspace, at the expense of losing manifest higher-dimensional Lorentz invariance [51, 52, 53, 54]. In this way, only a minimal $N=1$ subgroup of the extended higher-dimensional supersymmetry will be manifest, but this turns out to be enough for our purposes.

In fact, writing higher-dimensional supersymmetric theories in term of $N=1$ superfields not only simplifies loop computations, but makes it also easier to write down supersymmetric couplings between bulk and brane fields. As explained in ref. [55], the way of doing this is to group the higher-dimensional supermultiplets into subsets that transform under an $N=1$ subgroup of the full higher-dimensional supersymmetry. Brane couplings can then be written using the known 4d $N=1$ supersymmetric couplings. Splitting higher-dimensional multiplets in different $N=1$ superfields does just that, without having to look explicitly at the higher-dimensional supersymmetry algebra. Also, the couplings in component form often involve ambiguous products of δ functions that arise when auxiliary fields are integrated out. Using superfields this problem is avoided, because auxiliary fields are never integrated out. The drawback of the superfield approach is that, since higher-dimensional Lorentz invariance and supersymmetry are not manifest, one has to more or less guess the right Lagrangian, and check that it reproduces the correct higher-dimensional Lagrangian in component.

This program has been carried out for linearized 5d supergravity in *flat* space in ref. [54], and the resulting formalism has been successfully applied in ref. [24] to compute gravitational quantum corrections in orbifold models, with results that agree with those derived in ref. [23] by studying a particular component of the corresponding superspace effective operators. In this section, we will briefly review the approach of refs. [54, 24] for the case of a flat extradimension, and then generalize it to the case of a warped extradimension.

2.3.1 Bulk lagrangian for a flat space

The propagating fields of 5d supergravity consist of the graviton h_{MN}, the graviphoton B_M and the gravitino Ψ_M, which can be decomposed into two two-component Weyl spinors ψ_M^+ and ψ_M^-. These fields can be embedded into a real superfield V_m, a complex general superfield Ψ_α, and two chiral superfields $\mathscr{T}$ and Σ, according to the following schematic structure:

$$
\begin{aligned}
V_m &= \theta\sigma_n\bar{\theta}(h_{mn}-\eta_{mn}h)+\bar{\theta}^2\theta\psi_m^+ +\cdots\,, && (2.29)\\
\Psi_\alpha &= \bar{\theta}\bar{\sigma}^m\,(B_m+ih_{my})+\theta\sigma_m\bar{\theta}\psi_m^- +\bar{\theta}^2\psi_y^+ +\cdots\,, && (2.30)\\
\mathscr{T} &= h_{yy}+iB_y+\theta\psi_y^- +\cdots\,, && (2.31)\\
\Sigma &= s+\cdots\,. && (2.32)
\end{aligned}
$$

The dots denote higher-order terms involving additional fields, which are either genuine auxiliary fields or fields that are a priori not, but eventually turn out to be, non-propagating. We also need to introduce a real superfield P_Σ acting as a prepotential for the chiral conformal compensator Σ: $\Sigma=-1/4\bar{D}^2P_\Sigma$. This introduces yet more non-propagating fields.

Using the above fields, it is possible to construct in an unambiguous way a linearized theory that is invariant under infinitesimal transformations of all the local symmetries characterizing a 5d supergravity theory on an interval. These linearized gauge transformations consist of the usual 4d superdiffeomorphisms, which are parametrized by a general complex superfield L_α, and the additional transformations completing these to 5d superdiffeomorphims, which are parametrized by a chiral multiplet Ω. The corresponding linearized gauge transformations of the superfields introduced above are given by

$$
\begin{aligned}
\delta V_m &= -\frac{1}{2}\bar{\sigma}_m^{\alpha\dot{\alpha}}\,(\bar{D}_{\dot{\alpha}}L_\alpha-D_\alpha\bar{L}_{\dot{\alpha}})\\
\delta\Psi_\alpha &= \partial_yL_\alpha-\frac{1}{4}D_\alpha\Omega\\
\delta P_\Sigma &= D^\alpha L_\alpha+\text{h.c}\\
\delta\mathscr{T} &= \partial_y\Omega\,. && (2.33)
\end{aligned}
$$

As usual, L_α also contains conformal transformations that extend the 4d super-Poincaré group to the full 4d superconformal group, but these extra symmetries are fixed by gauging away the compensator multiplet Σ.

The Lagrangian for linearized 5d supergravity in flat space can be constructed by writing the most general Lagrangian that is invariant under the above linearized gauge transformations. This fix the Lagrangian up to one unknown constant that can be determined by imposing that its component form be invariant under 5d Lorentz transformations. The result is

$$\mathscr{L} = M_5^3 \int d^4\theta \, \Big\{ \frac{1}{2} V^m K_{mn} V^n - \frac{1}{3}\Sigma^\dagger\Sigma + \frac{2i}{3}\left(\Sigma - \Sigma^\dagger\right)\partial^m V_m$$
$$- \frac{1}{2}\left[\partial_y V_{\alpha\dot\alpha} - \left(\bar D_{\dot\alpha}\Psi_\alpha - D_\alpha \bar\Psi_{\dot\alpha}\right)\right]^2 + \frac{1}{4}\left[\partial_y P_\Sigma - \left(D^\alpha \Psi_\alpha + \bar D_{\dot\alpha}\bar\Psi^{\dot\alpha}\right)\right]^2$$
$$- \frac{1}{2}\left[\mathscr{T}^\dagger\left(\Sigma + 2i\partial_m V^m\right) + \text{h.c.}\right]\Big\} , \tag{2.34}$$

where

$$K_{mn} = \frac{1}{4}\eta_{nm} D^\alpha \bar D^2 D_\alpha + \frac{1}{24}\bar\sigma_m^{\dot\alpha\alpha}\bar\sigma_n^{\dot\beta\beta}\left[D_\alpha, \bar D_{\dot\alpha}\right]\left[D_\beta, \bar D_{\dot\beta}\right] + 2\partial_m\partial_n \, . \tag{2.35}$$

The first line of (2.34) is the usual linearized supergravity Lagrangian. To obtain the component Lagrangian, one choose a suitable “Wess-Zumino” gauge, and eliminates all the auxiliary fields. By doing so, one correctly reproduces the linearized Lagrangian of 5d supergravity, plus some extra fields that do not propagate but have the dimensionality of propagating fields [54]. These superfluous fields will also appear in the warped Lagrangian. The above construction can be generalized to the orbifold $S_1/\mathbf{Z}_2$ in a straightforward way, by assigning a definite $\mathbf{Z}_2$ parity to each multiplet: V_m, Σ and $\mathscr{T}$ are even, whereas Ψ_α is odd.

The 5d Lagrangian (2.34) can be written in a physically more transparent form by using the complete set of projectors that define the different orthogonal components with superspin 0, 1/2, 1 and 3/2 of the real superfield V_m. These are defined as

$$\Pi_0^{mn} = \Pi_L^{mn} P_C \, , \tag{2.36}$$
$$\Pi_{1/2}^{mn} = \frac{1}{48}\frac{1}{\Box}\sigma^m_{\alpha\dot\alpha}\sigma^n_{\beta\dot\beta}\left[D_\alpha, \bar D_{\dot\alpha}\right]\left[D_\beta, \bar D_{\dot\beta}\right] + \Pi_L^{mn} P_T + \frac{1}{3}\Pi_0^{mn} \, , \tag{2.37}$$
$$\Pi_1^{mn} = \Pi_T^{mn} P_C \, , \tag{2.38}$$
$$\Pi_{3/2}^{mn} = -\frac{1}{48}\frac{1}{\Box}\sigma^m_{\alpha\dot\alpha}\sigma^n_{\beta\dot\beta}\left[D_\alpha, \bar D_{\dot\alpha}\right]\left[D_\beta, \bar D_{\dot\beta}\right] + \eta^{mn} P_T - \Pi_L^{mn} + \frac{2}{3}\Pi_0^{mn} \, , \tag{2.39}$$

in terms of the transverse and chiral projectors on vector superfields, which are given by

$$P_T = -\frac{1}{8}\frac{D^\alpha \bar D^2 D_\alpha}{\Box} \, , \quad P_C = \frac{1}{16}\frac{D^2\bar D^2 + \bar D^2 D^2}{\Box} \, , \tag{2.40}$$

and the transverse and longitudinal projectors acting on vector indices, given by

$$\Pi_T^{mn} = \eta_{mn} - \frac{\partial_m\partial_n}{\Box} \, , \quad \Pi_L^{mn} = \frac{\partial_m\partial_n}{\Box} \, . \tag{2.41}$$

The kinetic operator (2.35) can then be written as

$$K^{mn} = -2\Box\left(\Pi_{3/2}^{mn} - \frac{2}{3}\Pi_0^{mn}\right) , \tag{2.42}$$

Using the above complete set of superspin projectors, we can split the field V_m into 4 orthogonal parts V_0, $V_{1/2}$, V_1 and $V_{3/2}$. The first three transform non-trivially under local super-diffeomorphism, but not the last one, which is invariant. The Lagrangian (2.34) can then be equivalently rewritten as

$$\begin{aligned}\mathscr{L} = M_5^3 \int d^4\theta \, \Big\{ & -V^m_{3/2}(\Box + \partial_y^2) V_{3/2m} - \frac{2}{3}\Big[\partial_m V_0^m - \frac{i}{2}(\Sigma - \Sigma^\dagger)\Big]^2 \\ & -\frac{1}{2}\Big[\partial_y\big(V_0^{\dot\alpha\alpha} + V_{1/2}^{\dot\alpha\alpha} + V_1^{\dot\alpha\alpha}\big) - \big(\bar D_{\dot\alpha}\Psi_\alpha - D_\alpha \bar\Psi_{\dot\alpha}\big)\Big]^2 \\ & +\frac{1}{4}\Big[\partial_y P_\Sigma - \big(D^\alpha\Psi_\alpha + \bar D_{\dot\alpha}\bar\Psi^{\dot\alpha}\big)\Big]^2 + i\big(\mathscr{T} - \mathscr{T}^\dagger\big)\Big[\partial_m V_0^m - \frac{i}{2}(\Sigma - \Sigma^\dagger)\Big]\Big\} \, .\end{aligned} \tag{2.43}$$

We can see clearly in this language why the compensator is needed. The kinetic Lagrangian for the gauge-invariant component $V^m_{3/2}$ is non local, due to the singular form of $\Pi^{mn}_{3/2}$. This non-local part is canceled by a similar non-local part coming from the kinetic Lagrangian of the gauge-variant component V^m_0. The non-trivial variation under gauge transformation of this term is compensated by that of Σ. Therefore the kinetic term of linearized 4d supergravity, the first line of eq. (2.43), decomposes as the sum of two invariant terms respectively of maximal (3/2) and minimal (0) *superspin*. This is fully analogous to the situation in ordinary Einstein gravity where the linearized kinetic term decomposes as the sum of spin 2 and 0. Notice also that like in Einstein gravity $V^m_{3/2}$ being the only component of maximal superspin does not mix to any other combination.

We can verify that the correct 4d $N = 1$ effective Lagrangian is obtained for the zero modes, by taking the even fields to depend only on the four dimensional coordinates and integrating over the extradimension with radius R. To be precise, we use a hat to distinguish the 4d zero mode of each field from the corresponding 5d field itself. The result is:

$$\begin{aligned}\mathscr{L}_{\text{eff}} = 2\pi R M_5^3 \int d^4\theta \, \Big\{ & -\hat V^m_{3/2}(\Box)\hat V_{3/2m} - \frac{2}{3}\Big[\partial_m \hat V_0^m - \frac{i}{2}(\hat\Sigma - \hat\Sigma^\dagger)\Big]^2 \\ & + i\big(\hat{\mathscr{T}} - \hat{\mathscr{T}}^\dagger\big)\Big[\partial_m \hat V_0^m - \frac{i}{2}(\hat\Sigma - \hat\Sigma^\dagger)\Big]\Big\} \, .\end{aligned} \tag{2.44}$$

It can be verified (trivially for the V_m-independent terms) that this is indeed the quadratic expansion of the 4*d* supergravity Lagrangian, which can be written in terms of the 4d conformal compensator $\phi = \exp(\hat\Sigma/3)$ and the full 4d radion field $T = \pi R(1 + \hat{\mathscr{T}})$ as

$$\mathscr{L}_{\text{eff}} = M_5^3 \int d^4\theta \, \big(T + T^\dagger\big)\phi^\dagger\phi \, . \tag{2.45}$$

where in this expression, the $d^4\theta$ integration is in fact an abbreviated notation for taking the D term of this expression in a covariant manner. In particular, factors of the metric should be included. This result agrees with what was found in [49].

2.3.2 Bulk Lagrangian for warped space

We now turn our attention to the case of warped space. AdS_5 is not a solution of the ordinary, ungauged supergravity Lagrangian, which does not admit a cosmological constant. To have a cosmological constant term, a $U(1)_R$ subgroup of the $SU(2)_R$ symmetry must be gauged by the graviphoton. Because we will restrict ourselves to the quadratic Lagrangian of supergravity, the gauging of the $U(1)_R$ cannot be seen in our formalism. We assume a fixed background defined in eq. (1.13). We then want to write the quadratic Lagrangian for the fluctuations around that background in term of 4d superfields. To show that this is indeed possible, we need to re-examine the logic.

We are interested in the quadratic Lagrangian for supergravity in five dimensions. At the local level there are two supersymmetries. However the boundary condition on S_1/Z_2 are such that globally there remains at most one supersymmetry, regardless of there being 5d curvature. As it has been discussed in several papers the locally supersymmetric RS model preserves one global supersymmetry Q^G_α. Technically that means that there exists one killing spinor ψ_K over the RS background. In bispinor notation we have

$$\psi_K = \begin{pmatrix} e^{-\sigma/2}\eta \\ 0 \end{pmatrix} \tag{2.46}$$

where η is a constant Weyl spinor. Indicating by $Q^L \equiv (Q^{L\,\alpha}_2, \bar{Q}^L_{1\dot\beta})$ the bispinor generator of the local 5d supersymmetries, we have that the global symmetry Q^G_α is just defined by

$$\psi_K \bar{Q}^L = e^{-\sigma/2}\eta^\alpha Q^L{}_{1\alpha} \equiv \eta^\alpha Q^G_\alpha. \tag{2.47}$$

We can realize Q^G and the rest of the global 4d superPoincarè group ($P_\mu = i\partial_\mu$ plus Lorentz boosts) over ordinary flat 4d superspace

$$Q^G_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^a_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\delta_a{}^\mu\partial_\mu\ , \tag{2.48}$$

$$Q^G_{\dot\alpha} = -\frac{\partial}{\partial\theta^{\dot\alpha}} + i\theta^\alpha\sigma^a_{\alpha\dot\alpha}\delta_a{}^\mu\partial_\mu\ . \tag{2.49}$$

In this realization of our field space the fifth coordinate y is just a label upon which our 4d superfields $S(\theta,\bar\theta,x^\mu,y)$ depend. Of course 5d covariance is never manifest in this formulation of the theory and the correct action is obtained via the explicit dependence of the superspace lagrangian on y and ∂y. By expanding $S = \sum_n S_n(x,y)\theta^n$ we identify $S_n(x,y)$ with the local 5d fields. However our global supersymmetry knows little about the local 5d geometry, so that in general the S_n are not normalized in a way that makes 5d covariance manifest. This is obviously not a problem: the correct normalization (as well as the correct covariant derivative structure) can always be obtained by local redefinitions of the S_n by powers of the warp factor [56, 57] We can figure out the right rescaling reproducing the canonical fields by considering the normalization of the supercharge. From Eq. (2.47) we have that the local supersymmetry is $Q^L = e^{\sigma/2}Q^G$. Then by defining a "local" superspace coordinate $\tilde\theta = e^{-\sigma/2}\theta$ we can write

$$Q^L_\alpha = \frac{\partial}{\partial\tilde\theta^\alpha} - i\sigma^a_{\alpha\dot\alpha}\bar{\tilde\theta}^{\dot\alpha}e_a{}^\mu\partial_\mu \tag{2.50}$$

where the flat veilbein δ_a^μ has been substituted by the curved one $e_a^\mu = e^\sigma \delta_a^\mu$. The presence of the vielbein, not surprisingly, shows that Q^L is covariant and realizes the local supersymmetry algebra

$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = -2i\sigma^a_{\alpha\dot\alpha} e_a{}^m \partial_m. \tag{2.51}$$

Therefore if we parametrize our superfields in terms of the "local" $\tilde\theta$ as opposed to the global θ the field coefficients should correspond to the local canonical fields. Using $S = \sum_n S_n \theta^n \equiv \sum_n S_n e^{n\sigma/2} \tilde\theta^n$ we conclude that

$$S_n(x,y) = e^{-n\sigma/2} S_n(x,y)^{local}. \tag{2.52}$$

Notice that we have been a little sloppy here: the superfields coefficients S_n involve in some cases 4d derivatives ∂_μ. By eq. (2.50), the change of coordinates $\theta = e^{\sigma/2}\tilde\theta$ corresponds to $\partial_\mu \to \partial_a \equiv e_a^\mu \partial_\mu$. For instance in the case of a chiral superfield we have

$$\begin{aligned} \phi &= e^{-i\bar\theta\sigma^a\bar\theta\delta_a^\mu\partial_\mu}\left(\varphi + \chi\theta + F\theta^2\right) = & (2.53)\\ &= e^{-\bar i\theta\sigma^a\bar{\tilde\theta} e_a^\mu \partial_\mu}\left(\varphi + \chi e^{\sigma/2}\tilde\theta + e^\sigma F\tilde\theta^2\right). & (2.54)\\ && (2.55) \end{aligned}$$

Our conclusions are not affected. One final question: why are then we not working right away with the $\tilde\theta$ coordinates? The reason is that Q^L does not commute with ∂_5 [6] so that $\partial_5 S$ is not a superfield over $\tilde\theta$. We could define a supercovariant D_5 derivative, but we find it more convenient to work with global, flat, superspace.

Now that we know that it is possible to write the desired Lagrangian in term of standard $N = 1$ superfields, we need to examine the gauge symmetry that this Lagrangian should possess. Parametrizing the fluctuations around the background as

$$ds^2 = e^{-2\sigma}\left(\eta_{mn} + h_{mn}\right) dx^m dx^n + 2e^{-\sigma} h_{my} dx^m dy + (1 + h_{yy}) dy^2 \ , \tag{2.56}$$

it follows that the linearized transformations law under general coordinate transformations are given by

$$\begin{aligned} \delta h_{mn} &= \partial_m \xi_n + \partial_n \xi_m - 2\sigma' \eta_{mn} \xi_y \ , & (2.57)\\ \delta h_{my} &= e^{-\sigma} \partial_y \xi_m + e^\sigma \partial_m \xi_y \ , & (2.58)\\ \delta h_{yy} &= \partial_y \xi_y \ . & (2.59) \end{aligned}$$

Comparing with the flat case, we see that the warping is responsible for a new term proportional to σ' in the transformation law for h_{mn}.

The embedding of component fields into superfields can be done as in the flat case, except that we need to introduce in this case a real prepotential $P_\mathscr{T}$ for $\mathscr{T}$ as well, in such

[6] Notice $Q^L = e^{\sigma(y)/2} Q^G$ is explicitly y dependent.

a way that $\mathscr{T} = -1/4\bar{D}^2 P_{\mathscr{T}}$. The transformation laws can then be written in terms of superfields, after introducing a prepotential P_Ω also for Ω, as:

$$\delta V_m = -\frac{1}{2}\bar{\sigma}_m^{\dot{\alpha}\alpha}\left(\bar{D}_{\dot{\alpha}}L_\alpha - D_\alpha \bar{L}_{\dot{\alpha}}\right) , \tag{2.60}$$

$$\delta\Psi_\alpha = e^{-\sigma}\partial_y L_\alpha - \frac{1}{4}e^{\sigma}D_\alpha\Omega , \tag{2.61}$$

$$\delta P_\Sigma = D^\alpha L_\alpha - 3\sigma' P_\Omega , \tag{2.62}$$

$$\delta P_{\mathscr{T}} = \partial_y P_\Omega . \tag{2.63}$$

From the last two expressions, it follows that:

$$\delta\Sigma = -\frac{1}{4}\bar{D}^2 D^\alpha L_\alpha - 3\sigma'\Omega , \tag{2.64}$$

$$\delta\mathscr{T} = \partial_y\Omega . \tag{2.65}$$

Note that the new term proportional to σ' in the transformation law for h_{mn} is encoded in superfield language in a new term in the transformation law for Σ. This is possible because, in addition to general coordinate invariance, the transformations parametrized by L_α also include Weyl and axial transformations. These extra conformal transformations can be fixed by setting the lowest component s of the compensator Σ to 0. But the subgroup of transformations that preserve this gauge choice involves scale transformations that are correlated with diffeomorphism and induce the appropriate extra term in eq. (2.57). To show this more precisely, let us consider the transformation laws of h_{mn} and s under diffeomorphisms with real parameters ξ_M and complexified Weyl plus axial transformations with complex parameter λ, as implied by eqs. (2.60) and (2.64):

$$\delta h_{mn} = \partial_m\xi_n + \partial_n\xi_m - \frac{2}{3}\partial_m\xi^m + \frac{1}{6}\eta_{mn}\left(\lambda + \lambda^*\right) , \tag{2.66}$$

$$\delta s = 2\partial_m\xi^m - 6\sigma'\xi_y - \lambda . \tag{2.67}$$

As anticipated we can now use the Weyl and axial symmetries associated to λ to set s to 0. To preserve that gauge choice, however, diffeomorphisms must then be accompanied by a suitable Weyl transformations with parameter

$$\lambda + \lambda^* = 4\partial_m\xi^m - 12\sigma'\xi_y . \tag{2.68}$$

Plugging this expression back into (2.66), we find the net transformation of the graviton under a diffeomorphism after the conformal gauge-fixing reproduces indeed eq. (2.57).

It is now straightforward to construct a superfield Lagrangian that is invariant under the transformations (2.60)–(2.65) and reduces to (2.34) in the flat limit. It has the following expression:

$$\mathscr{L} = M_5^3 \int d^4\theta\, e^{-2\sigma} \Big\{ -V_{3/2}^m \left(\Box + e^{2\sigma}\partial_y e^{-4\sigma}\partial_y\right) V_{3/2\,m} - \frac{2}{3}\Big[\partial_m V_0^m - \frac{i}{2}(\Sigma - \Sigma^\dagger)\Big]^2$$
$$-\frac{1}{2}\Big[e^{-\sigma}\partial_y\big(V_0^{\dot\alpha\alpha} + V_{1/2}^{\dot\alpha\alpha} + V_1^{\dot\alpha\alpha}\big) - \big(\bar D_{\dot\alpha}\Psi_\alpha - D_\alpha\bar\Psi_{\dot\alpha}\big)\Big]^2$$
$$+\frac{1}{4}\Big[e^{-\sigma}\big(\partial_y P_\Sigma + 3\sigma' P_{\mathscr{T}}\big) - \big(D^\alpha\Psi_\alpha + \bar D_{\dot\alpha}\bar\Psi^{\dot\alpha}\big)\Big]^2$$
$$+ i\big(\mathscr{T} - \mathscr{T}^\dagger\big)\Big[\partial_m V_0^m - \frac{i}{2}(\Sigma - \Sigma^\dagger)\Big]\Big\}\,. \tag{2.69}$$

There is one important remark to make about this Lagrangian: it possesses an extra (accidental) local invariance in addition to those we employed to derive it

$$\delta\Psi_\alpha = 3\sigma' e^{-\sigma} W_\alpha \tag{2.70}$$
$$\delta P_{\mathscr{T}} = D^\alpha W_\alpha + D_{\dot\alpha} W^{\dot\alpha} \tag{2.71}$$

where W_α is a chiral field. This new symmetry is associated to a redundancy in the parametrization of $\mathscr{T}$ by a prepotential $P_{\mathscr{T}}$. Indeed $P_{\mathscr{T}}$ shifts by a linear multiplet so that using W_α we can gauge away the newly introduced components of $P_{\mathscr{T}}$. By this invariance of the quadratic action some combinations of fields have no kinetic term. In order for our expansion to make sense when going to non-linear order it is important to demand full invariance under this new transformation.

The component form of the Lagrangian is reported in appendix A. It reproduces the correct component Lagrangian for supergravity on a slice of AdS_5 at the linearized level. The well known derivation of the low energy effective theory and KK decomposition then follows. It is however instructive to derive these results in terms of superfields.

Let us construct the zero mode superfield action first. First of all, Ψ_α is Z_2 odd so that it does not have zero modes. Second, notice that if we choose $V^m(x,y) \equiv \hat V^m(x)$ the mass terms, involving ∂_y cancel out. On the other hand, the mass term involving P_Σ and $P_{\mathscr{T}}$ in the third line is non vanishing for y independent field configurations. One possible parametrization of the zero modes for which this term vanishes altogether is[7]

$$\begin{aligned} P_{\mathscr{T}}(x,y) &\equiv \hat P_{\mathscr{T}}(x) & \quad (\mathscr{T}(x,y) &\equiv \hat{\mathscr{T}}(x)) \\ P_\Sigma(x,y) &\equiv \hat P_\Sigma(x) - 3\sigma(y)\hat P_{\mathscr{T}}(x) & \quad (\Sigma(x,y) &\equiv \hat\Sigma(x) - 3\sigma(y)\hat{\mathscr{T}}). \end{aligned} \tag{2.72}$$

Notice that the conformal compensator Σ depends on y precisely like the conformal factor of the metric does in the zero mode parametrization for the bosonic RS1 model [58]. In order to write the effective action in compact form it is useful to notice that $\partial_m V_0^m = \chi + \chi^\dagger$ and then form the two combinations

$$\Sigma_A \equiv \Sigma + 2i\chi \qquad \Sigma_B \equiv \Sigma - 2i\chi \tag{2.73}$$

[7] One can check that, by using the gauge freedom associated to L_α and W_α, $\hat P_{\mathscr{T}}(x)$ and $\hat P_\Sigma(x)$ can be chosen to be purely chiral + antichiral (i.e no linear superfi eld component) while keeping $\psi_\alpha = 0$.

of which only Σ_A is gauge invariant. The zero mode action, with the second and third line in eq. (2.69) vanishing, depends only on Σ_A. Defining zero modes in analogy with the above

$$\Sigma_A(x,y) \equiv \hat{\Sigma}(x) - 3\sigma(y)\hat{\mathscr{T}}(x) + 2i\hat{\chi}(x) \equiv \hat{\Sigma}_A(x) - 3\sigma(y)\hat{\mathscr{T}}(x) \tag{2.74}$$

the superspin zero 5d action is simply

$$\frac{1}{2\sigma'}\frac{\partial}{\partial y}\left[\frac{e^{-2\sigma}}{3}\left(\hat{\Sigma}_A^\dagger - 3\sigma\hat{\mathscr{T}}^\dagger\right)\left(\hat{\Sigma}_A - 3\sigma\hat{\mathscr{T}}\right)\right] \tag{2.75}$$

showing that the geometry of the boundaries is what matters in the low energy effective action. The quadratic action for the zero modes is then

$$\mathscr{L}_{\rm eff} = \frac{M_5^3}{k}\int d^4\theta \left\{-\left(1-e^{-2\pi kR}\right)\hat{V}^m_{3/2}\Box\hat{V}_{3/2m}\right. \tag{2.76}$$

$$\left. -\frac{1}{3}\left[-\hat{\Sigma}_A^\dagger\hat{\Sigma}_A + e^{-2\pi kR}\left(\hat{\Sigma}_A^\dagger - 3\pi kR\hat{\mathscr{T}}^\dagger\right)\left(\hat{\Sigma}_A - 3\pi kR\hat{\mathscr{T}}\right)\right]\right\} \tag{2.77}$$

By expliciting the dependence of $\hat{\Sigma}_A$ on V_m we find, as it should, that this Lagrangian is local. Notice that with the identification $\phi = \exp(\hat{\Sigma}/3)$ and $T = \pi R(1+\hat{\mathscr{T}})$ the scalar part agrees with the quadratic expansion of the full non linear result which was inferred by general arguments in ref. [25][8]

$$\mathscr{L}_{\rm eff} = \frac{M_5^3}{k}\int d^4\theta \left(1-e^{-k(T+T^\dagger)}\right)\phi^\dagger\phi\ . \tag{2.78}$$

Similarly we can study the KK spectrum. For doing so it is convenient to work in the gauge $\Psi_\alpha = 0$. This makes it immediately evident that $V_{1/2}$ and V_1 do not propagate

[8] Working at the linearized level Re $t \equiv \mathscr{T}(\theta=\bar{\theta}=0) + \mathscr{T}(\theta=\bar{\theta}=0)^\dagger$ in principle coincides with $h_{55} \equiv \sqrt{g_{55}} - 1$ only up to higher order terms. However one can argue that by a holomorphic field redefinition $\mathscr{T} \to \mathscr{T}' = f(T)$ it should always be possible to choose Re $t' \equiv \sqrt{g_{55}} - 1$. The reason is that there exist covariant F-terms, like the gauge kinetic term, for which the right geometric dependence on g_{55} could only be obtained by holomorphic field redefinitions. Let us then choose $\mathscr{T}$ such that Re $t = \sqrt{g_{55}} - 1$. Now, focusing on the constant mode of $\sqrt{g_{55}}$, we know that the low energy Lagrangian can only depend on it via the covariant combination $R\sqrt{g_{55}}$ equaling the physical length of the 5th dimension. This is not yet enough to fully fix the dependence on $\mathscr{T}$. We need to use the constraints on the dependence on Im $t \propto A_5$ the graviphoton 5th component. The VEV of the low energy Kähler potential corresponds to the effective 4d Planck scale of the usual RS1 model, which does not depend on other bulk fields than the radion. In particular it does not dependent on gauge fields like the graviphoton. This fixes completely the dependence on $\mathscr{T}$ to be obtained by the simple substitution $2R \to R(2+\hat{\mathscr{T}}+\mathscr{T}^\dagger)$, compatibly with our results. The dependence on A_5 is actually constrained even at the quantum level by the presence of an accidental (gauge) symmetry $A_5 \to A_5 + const.$ of minimal 5d supergravity on S_1/Z_2. The point is that the graviphoton appears in covariant derivative via a Z_2-odd charge $\partial_5 \to \partial_5 + iq\varepsilon(y)A_5$: basically the graviphoton is a Z_2 odd gauge field used to gauge an even symmetry. It is then evident that a constant $A_5 = a$ configuration can be gauged away by the a gauge rotation with parameter $\alpha = a\sigma(y)/(\pi kR)$.

while $V_{3/2}$ has the KK decomposition of the graviton in RS1. Furthermore by using P_Ω and W_α we can conveniently choose $P_{\mathscr{T}}$ to have the form

$$P_{\mathscr{T}}(x,y) = e^{2\sigma} P^0_{\mathscr{T}}(x). \tag{2.79}$$

where, moreover, $D^2 D^\alpha P_{\mathscr{T}}(x)^0 = 0$. In practice we have eliminated all the modes in $P_{\mathscr{T}}$ apart from a chiral radion mode. Next by using a residual freedom $L_\alpha \equiv S_\alpha(x)$ with S_α chiral, under which ψ_α and V_m are unaffected, we can eliminate the linear superfield mode in P_Σ which is constant over y. Therefore in the superspin 1/2 sector of P_Σ, $P_{\mathscr{T}}$ only the non-trivial KK modes $P_\Sigma^{(n)}$ ($n \neq 0$) are left. Like for $V_{1/2}$ and V_1 their kinetic lagrangian is a simple quadratic term $\propto m_n^2 (P_\Sigma^{(n)})^2$ with trivial mass shell condition $P_\Sigma^{(n)} = 0$. In what follows we can then concentrate on the superspin zero components and work directly with the chiral fields Σ and $\mathscr{T}$. By eq. (2.79) we have $\mathscr{T}(x,y) = e^{2\sigma}\hat{\mathscr{T}}_0(x)$. It is also convenient to parametrize Σ as

$$\Sigma(x,y) = -\frac{3}{2} e^{2\sigma} \hat{\mathscr{T}}_0(x) + \tilde{\Sigma}(x,y) \tag{2.80}$$

so that for $\Sigma_{A,B}$ (cfr. eq. (2.73)) we have

$$\Sigma_A(x,y) = -\frac{3}{2} e^{2\sigma} \mathscr{T}_0(x) + \tilde{\Sigma}_A(x,y) \qquad \Sigma_B(x,y) = -\frac{3}{2} e^{2\sigma} \mathscr{T}_0(x) + \tilde{\Sigma}_B(x,y). \tag{2.81}$$

With this parametrization the Lagrangian for the chiral fields becomes

$$\int d^4\theta dy \left\{ \frac{3}{4} \mathscr{T}_0 \mathscr{T}_0^\dagger e^{2\sigma} - \frac{1}{3} \tilde{\Sigma}_A \tilde{\Sigma}_A^\dagger e^{2\sigma} + \frac{e^{-4\sigma}}{4} \left(\partial_y \tilde{\Sigma}_B \frac{1}{\Box} \partial_y \tilde{\Sigma}_A^\dagger + \text{h.c.} \right) \right\}. \tag{2.82}$$

This parametrization makes manifest that the non-zero KK modes of $\tilde{\Sigma}_B$ act as Lagrange multipliers for the modes of $\tilde{\Sigma}_A$. The only physical modes left are therefore $\mathscr{T}_0$ along with the zero mode of $\tilde{\Sigma}_A$: they represent an alternative parametrization of the zero modes, one in which the radion does not mix kinetically with the 4d graviton. This is the superfield analogue of the radion parametrization discussed in ref. [59].

2.3.3 Boundary Lagrangians

The only interactions that are needed for our calculation are those between brane and bulk fields. At the fixed points, Ψ_α vanishes, V_m and Σ undergo the same transformations of linearized 4d supergravity (remember that Ω is odd and vanishes at the boundaries) and finally $\mathscr{T}$ is the only field transforming under Ω. By this last property $\mathscr{T}$ cannot couple to the boundary, so that only V_m and Σ can couple and they must do so precisely like they do in 4d supergravity (see [24]). What is left are the 4d superdiffeomorphisms of the boundaries. The presence of warping shows up in the boundary Lagrangian via the suitable powers of the warp factor. These are easy to evaluate according to our discussion in the previous section. Using locally inertial coordinates $\tilde{x}$ and $\tilde{\theta}$ no power of the warp factor should appear in the invariant volume element $d^4\tilde{x}d^4\tilde{\theta}$ for the Kähler potential and $d^4\tilde{x}d^2\tilde{\theta}$ for the superpotential. From ordinary $N=0$ RS we know that $d^4\tilde{x} = e^{-4\sigma}d^4x$ where x are the global coordinates, while from the previous section we have learned that $\tilde{\theta} = e^{-\sigma/2}\theta$. Indicating by $\sigma(i)$ $(i=0,1)$ the warp function at the two boundaries we conclude that for the localized action the right warp factors multiplying it are $e^{-2\sigma(i)}$ and $e^{-3\sigma(i)}$ for respectively the Kähler and superpotential.

Let us consider a chiral superfields Φ_i localized at the i-th brane and with quadratic Kähler potential $\Omega_i = \Phi_i^\dagger\Phi$. As we already argued the coupling of Φ_i to V_m and Σ is the same as in ordinary 4d supergravity. For our purposes, since we are only interested in the 1-loop Kähler potential, it is sufficient to consider terms that are at most quadratic in V_m and Σ, with derivatives acting on at most one of Φ_i and $\Phi_i^\dagger$. The relevant part of the 4d boundary Lagrangian at $y=y_i$ is then given by

$$\mathscr{L}_i = \int d^4\theta e^{-2\sigma(i)}\left[\Phi_i^\dagger\Phi_i\left(1+\frac{\Sigma}{3}+\frac{\Sigma^\dagger}{3}+\frac{\Sigma\Sigma^\dagger}{9}\right)+\frac{2}{3}i\Phi_i^\dagger\overleftrightarrow{\partial_m}\Phi_iV^m-\frac{1}{6}\Phi_i^\dagger\Phi_iV^mK_{mn}V_n\right]. \tag{2.83}$$

To compute the effective Kähler potential we split the matter field into classical background $\bar{\Phi}_i$ and quantum fluctuation: $\Phi_i = \bar{\Phi}_i + \pi_i$. Since only the massive KK modes are relevant to compute the non-local, radius dependent part of the potential, it is useful to write the action in superspin components

$$\begin{aligned}\mathscr{L}_i = \int d^4\theta\sqrt{g_{4i}}\,\Big[&\bar{\Phi}_i^\dagger\bar{\Phi}_i+\pi_i^\dagger\pi_i+\frac{1}{3}\bar{\Phi}_i^\dagger\pi_i\Sigma_A^\dagger+\frac{1}{3}\bar{\Phi}_i\pi_i^\dagger\Sigma_A\\ &-\frac{1}{3}\bar{\Phi}_i^\dagger\bar{\Phi}_i\left(V_{3/2}^m\Box V_{3/2m}-\frac{1}{3}\Sigma_A^\dagger\Sigma_A\right)\Big].\end{aligned} \tag{2.84}$$

where the gauge invariant combination Σ_A has been defined before, and where we have neglected terms involving derivative of the background and interactions among the quantum fluctuations, as they do not affect the Kähler potential at 1-loop. Similarly, for the case of localized kinetic terms $M_i^2R(g_i)$, as there is no propagating π_i, we get only the second line in eq. (2.84) with $\Phi^\dagger\Phi \to -3M_i^2$ (where we use the same normalization as ref.[23]).

Finally by using our parametrization of the radion and compensator zero modes we can check that the boundary contribution to the low energy effective Lagrangian is just the linearized version of the full non linear result

$$\mathscr{L}_{0,1} = \phi^\dagger\phi\left\{\Omega_0(\Phi_0,\Phi_0^\dagger)+\Omega_1(\Phi_1,\Phi_1^\dagger)e^{-k(T+T^\dagger)}\right\}. \tag{2.85}$$

2.3.4 Gauge fixing

We are now a in position to set-up more concretely the calculation of gravitational quantum corrections to the 4d effective Kähler potential. We are only interested in the finite terms that depend on the radion and the matter chiral multiplets. These finite terms are those that cannot arise from local renormalization of the original tree level Lagrangian.

These uninteresting local renormalizations correspond to terms of the same form as eqs. (2.78,2.85), and as such do not lead to any brane to brane or radion to brane mediation of supersymmetry breaking. Therefore the calculable terms are those and only those that affect the mediation of supersymmetry breaking: they correspond to non-local effects from the 5d perspective. As we restrict our attention to the Kähler potential, we neglect all derivatives on the external fields. The supergraph calculation that needs to be done becomes then very similar to the calculation of the Coleman–Weinberg potential in a non-supersymmetric theory [60]. Similar superfield computations have already been done for the gauge corrections to the Kähler potential in 4d supersymmetric theories in [61].

Normally for doing these computations the most convenient procedure is to add a suitable gauge fixing and work in generalized R_ξ gauges. In the case at hand it turns out there is a much simpler approach The point is that the fields $V_{3/2}$ and Σ_A appearing in the boundary Lagrangian are already gauge invariant combinations. Their propagator is gauge independent so that in order to calculate it suffices to work with the simplest possibility: unitary gauge. We have already presented the Lagrangian in this gauge. In particular notice that for the non zero KK modes satisfying $\partial_y\tilde{\Sigma}_A \neq 0$ the structure of the kinetic matrix is

$$\left(\tilde{\Sigma}_A^\dagger, \Sigma_B^\dagger\right) \begin{pmatrix} -\frac{e^{-2\sigma}}{3} & \frac{e^{-4\sigma}}{4}\frac{\partial_y^2}{\Box} \\ \frac{e^{-4\sigma}}{4}\frac{\partial_y^2}{\Box} & 0 \end{pmatrix} \begin{pmatrix} \tilde{\Sigma}_A \\ \tilde{\Sigma}_B \end{pmatrix} \tag{2.86}$$

so that, inverting the matrix, we find that the KK propagators $\langle \tilde{\Sigma}^{(n)\dagger}\tilde{\Sigma}_A^{(m)}\rangle$ vanishes for $n,m \neq 0$. By the definition of $\tilde{\Sigma}_A$ in eq. (2.81) it follows that the KK components of Σ_A drop out of our computation. On the other hand, at the zero mode level we have two chiral modes, the radion $\mathscr{T}_0$ and $\tilde{\Sigma}_A(x,y) = \tilde{\Sigma}_A^{(0)}(x)$, whose kinetic Lagrangian is obtained by integrating eq. (2.82). It is straightforward to check that the resulting brane to brane propagator $\langle \Sigma_A(x,y=0)^\dagger \Sigma_A(x,y=\pi)\rangle$ vanishes because of a cancellation between the radion and $\tilde{\Sigma}_A^{(0)}$ contributions. This is only possible thanks to the ghostly nature of the second field: this superfield contains indeed the conformal mode of the 4d graviton. The propagators on each individual brane like for instance $\langle \Sigma_A(x,y=0)^\dagger \Sigma_A(x,0)\rangle$ do not vanish, but have no non-trivial volume dependence: they contribute to the localized UV divergences and to the IR singularities associated to these physical 4d fields. By simple dimensional analysis the latter effects do not contribute to the Kähler potential. Therefore Σ_A is not relevant to our computation and we need to focus just on $V_{3/2}$. The relevant Lagrangian is then

$$\mathscr{L}_{3/2} = \int d^4\theta \Big[-V_m \Pi_{3/2}^{mn} \big(e^{-2\sigma}\Box(1+\delta(y)\rho_0 + \delta(y-\pi)\rho_1) + \partial_y e^{-4\sigma}\partial_y\big) V_n \tag{2.87}$$

where $\rho_{0,1}$ are the boundary corrections to kinetic terms, including the effect of matter field VEVs. The crucial remark is that, due to the ubiquitous presence of the projector,*the spin structure factors out and the calculation reduces to that for a simple scalar* *field*. This is in agreement with the results found in ref. [23] for the flat case. For phenomenological application we will be interested in the case $\rho_i = M_i^2 - \Phi_i^\dagger \Phi_i/3$. Inverting the above kinetic term the superspin structure factors out in an overall projector

$$\left\langle V_m^{3/2}(x_1,y_1,\theta_1) V_n^{3/2}(x_2,y_1,\theta_2)\right\rangle = -\Pi_{3/2}^{mn}\delta^4(\theta_1-\theta_2)\Delta(x_1,y_1;x_2,y_2)\ , \qquad (2.88)$$

where Δ is the propagator of a real scalar field in a slice of AdS_5, in presence of boundary kinetic terms, defined by the equation

$$\left[e^{-2\sigma}\Box(1+\rho_0\delta(y)+\rho_\pi\delta(y-\pi))+\partial_y e^{-4\sigma}\partial_y\right]\Delta(x_1,y_1;x_2,y_2) = \delta^4(x_1-x_2)\delta(y_1-y_2)\,. \qquad (2.89)$$

This is agreement with the results found in ref. [23] for the flat case. By applying the methods of refs.[9] the 1-loop effective action can be written in a factorized form as

$$\Gamma_1 = -\frac{1}{2}\int d^5X d^5X' \delta^5(X-X') d^4\theta d^4\theta' \delta^4(\theta-\theta')\eta_{mn}\Pi_{3/2}^{mn}\delta^4(\theta-\theta')(\ln\Delta^{-1})(X,X') \qquad (2.91)$$

where $d^5X \equiv d^4xdy$. By using

$$\int d^4\theta\int d^4\theta'\,\delta^4(\theta-\theta')\,\eta_{mn}\Pi_{3/2}^{mn}\,\delta^4(\theta-\theta') = \int d^4\theta\frac{-4}{\Box}\,. \qquad (2.92)$$

and by expanding the scalar propagator Δ in its KK modes, the 1-loop kinetic function reads

$$\Delta\Omega_{1-loop} = -1\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\sum_n\frac{-4}{p^2}\ln(p^2+m_n^2). \qquad (2.93)$$

Apart from the $-4/p^2$ factor, arising from the superspace trace, this formula is just the Casimir energy of a scalar whose propagation is governed by eq. (2.89). From now on we will just need to concentrate on this toy scalar Lagrangian.

Notice that the dependence of the boundary interactions on just gauge invariant superspin $3/2$ and 0 component is a general result valid in any codimension, and not limited to 5d theories. What is specific of 5d is that the scalar component does not possess any propagating KK mode, and therefore cannot contribute to the genuinely calculable effects like the Casimir energy or brane to brane mediation. In higher codimensions this is no longer true, so that the scalar channel can in principle contribute. However it turns that, for a simple but remarkable property of eq. (2.84) this happens only for the Casimir energy but not for the terms that are quadratic in the matter fields like brane to brane or radion to brane. The reason is that eq. (2.84) inherits from the original quadratic matter

[9] This relation can be obtained by fi rst defi ning the trivial gaussian functional integral for a non propagating fi eld

$$\int DVe^{\int d^4xd^4\theta V_m V^m} = 1. \qquad (2.90)$$

The interesting case is then obtained by changing the trivial kinetic function to $\eta_{mn}\to\eta_{mn}+\Pi_{mn}^{3/2}(\Delta^{-1}-1)$. The resummation in perturbation theory of all the insertions of $\Pi^{3/2}$ leads to eq. (2.91) lagrangian a rescaling symmetry under which the compensator dependence can be fully absorbed by a redefinition of the matter field. In eq. (2.84) this rescaling amounts to the shift $\pi_i \to \pi_i - \Phi_i \Sigma_A/3$. Using this property, we can, for diagrams involving matter, do our computation by eliminating the compensator Σ (not Σ_A) first. After that we can work in a generalization of Landau gauge, where the $\langle VV \rangle$ is proportional to the $\Pi_{3/2}$ projector. In this case the diagrams involving cubic vertices cancel individually. This is in analogy with what happens for the calculation of the effective potential in $N = 0$ theories.

As we already know the gauge transformation $\delta V_{\alpha\dot{\alpha}} = D_\alpha \bar{L}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}} L_\alpha$ spans the subspace of components with superspin 0, 1/2 and 1. This means that L_α can be used to adjust the components V_0^m, $V_{1/2}^m$ and V_1^m, but not $V_{3/2}^m$. As a consequence, the most general acceptable gauge-fixing Lagrangian consists of a combination of quadratic terms for V_0^m, $V_{1/2}^m$ and V_1^m.

The class of gauge-fixing Lagrangian that needs to be added in order to reach the gauge where only $V_{3/2}^m$ propagates is given by the following expression:

$$\mathscr{L}_{\text{gf}} = \int d^4\theta e^{-2\sigma} \left[-\frac{1}{\xi} V_m \left(\eta^{mn} - \Pi_{3/2}^{mn} \right) V_n - \frac{2}{3} V_m \Pi_0^{mn} V_n \right] . \tag{2.94}$$

The part of the total Lagrangian that is quadratic in V_m then becomes:

$$\begin{aligned}\mathscr{L}_{\text{quad}} = \int d^4\theta \Big[& -V_m \Pi_{3/2}^{mn} (e^{-2\sigma}\Box + \partial_y e^{-4\sigma} \partial_y) V_n \\ & -\frac{1}{\xi} V_m (\eta^{mn} - \Pi_{3/2}^{mn}) e^{-2\sigma} \Box V_n \Big] .\end{aligned} \tag{2.95}$$

For $\xi \to 1$, and for a 4d theory where the extra superfields would be absent, this gauge-fixing would define the analog of the super-Lorentz gauge. Its form coincides with the one that was used in ref. [24], except for terms involving Σ, $\mathscr{T}$ and Ψ_α. For $\xi \to 0$, instead, this gauge-fixing defines the analog of the super-Landau gauge that we need. Indeed, it is clear that when ξ is sent to 0 only the $V_{3/2}^m$ component can propagate. Moreover, since $V_{3/2}^m$ does not couple to Ψ_α, Σ and $\mathscr{T}$, it is clear that the full $\langle V_m V_n \rangle$ is now *exactly* given by the left hand side of eq. (2.88).

2.4 One-loop effective potential

As demonstrated in the last section, the full 1-loop correction to the Kähler potential is encoded in the spectrum of a single real 5d scalar φ with Lagrangian

$$\mathscr{L} = \frac{1}{2} e^{-2\sigma(y)} \Big[- (\partial_\mu \varphi)^2 - e^{-2\sigma(y)} (\partial_y \varphi)^2 + \Big(\rho_0 \delta_0(y) + \rho_1 \delta_1(y) \Big) (\partial_\mu \varphi)^2 \Big] . \tag{2.96}$$

More precisely, see eq. (2.93), the effective 1-loop Kahler potential is obtained by inserting a factor $-4/p^2$ in the virtual momentum representation of the scalar Casimir energy. The superspace structure, at the end, in some way only counts the numbers of degrees

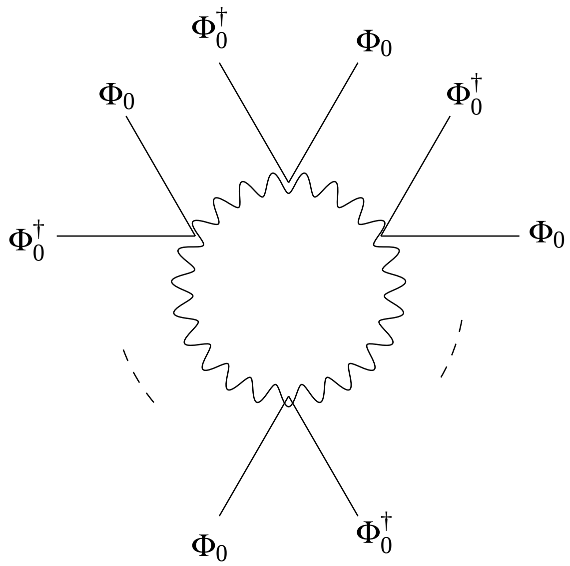


Figure 2.2: loop with n factors of $\Phi_0^\dagger \Phi_0$ attached to a V_m loop

of freedom via eq. (2.92). The factor 4 takes into account the multiplicity of bosonic and fermionic dergrees of freedom and the factor $1/p^2$ the fact that the Kähler potential determines the component effective action only after taking its D component. It is also understood that the circumference $2\pi R$ should be promoted to the superfield $T + T^\dagger$, and similarly the constants ρ_i should be promoted to the superfields $(-3M_i^2 + \Phi_i \Phi_i^\dagger)/(3M_5^3)$.

Let us now come to the computation. Along the lines followed in ref. [23] we find it convenient to start from $\rho_{0,\pi} = 0$ and to construct the full result by resumming the Feynman diagrams with all the insertions of $\rho_{0,\pi}$. The building blocks of this computation are the boundary-to-boundary propagators

$$\Delta_{ij}(p) = \sum_n e^{-\frac{3}{2}k|y_i|} e^{-\frac{3}{2}k|y_j|} \frac{\Psi_n(y_i)\Psi_n(y_j)}{p^2 + m_n^2} = e^{-\frac{3}{2}k|y_i|} e^{-\frac{3}{2}k|y_j|} \Delta(p, y_i, y_j) \tag{2.97}$$

with $y_{i,j} = 0, \pi R$. Here, and in what follows, Δ, $\Psi_n(y)$ and m_n denote the propagator, KK mode wave functions and masses for the scalar φ in the limit $\rho_0 = \rho_\pi = 0$. We work in mixed momentum-position space: momentum space along the non-compact directions, and configuration space along the 5th. The exponential factors have been introduced for later convenience. A second quantity relevant to compute the matter-independent Casimir energy is

$$Z(p) = \prod_n \left(p^2 + m_n^2\right) , \tag{2.98}$$

When going from the scalar φ to supergravity, Z will be the relevant object to compute the Kahler potential in the absence of both boundary matter and kinetic terms.

The explicit expressions for the above quantities are most conveniently written in terms of the functions $\hat{I}_{1,2}$ and $\hat{K}_{1,2}$, defined in terms of the standard Bessel functions $I_{1,2}$

and $K_{1,2}$ as

$$\hat{I}_{1,2}(x) = \sqrt{\frac{\pi}{2}}\sqrt{x}I_{1,2}(x)\,,\ \ \hat{K}_{1,2}(x) = \sqrt{\frac{2}{\pi}}\sqrt{x}K_{1,2}(x)\,. \tag{2.99}$$

These functions are elliptic generalizations of the standard trigonometric functions, and satisfy the relation

$$\hat{I}_1(x)\hat{K}_2(x) + \hat{K}_1(x)\hat{I}_2(x) = 1\,. \tag{2.100}$$

Their asymptotic behaviour at large argument $x \gg 1$ is given by:

$$\begin{aligned}\hat{I}_1(x) &\to e^{i\frac{\pi}{4}}\cosh\left(x - i\frac{\pi}{4}\right)\,,\ \ \hat{I}_2(x) \to e^{i\frac{\pi}{4}}\sinh\left(x - i\frac{\pi}{4}\right)\,,\\ \hat{K}_{1,2}(x) &\to e^{-i\frac{\pi}{4}}\left[\cosh\left(x - i\frac{\pi}{4}\right) - \sinh\left(x - i\frac{\pi}{4}\right)\right]\,.\end{aligned} \tag{2.101}$$

Similarly, their asymptotic behaviour at small argument $x \ll 1$ is given by:

$$\begin{aligned}\hat{I}_1(x) &\to \sqrt{\frac{\pi}{2}}\left[\frac{1}{2}x^{\frac{3}{2}} + \dots\right]\,,\ \ \hat{I}_2(x) \to \sqrt{\frac{\pi}{2}}\left[\frac{1}{8}x^{\frac{5}{2}} + \dots\right]\,,\\ \hat{K}_1(x) &\to \sqrt{\frac{2}{\pi}}\left[x^{-\frac{1}{2}} + \dots\right]\,,\ \ \hat{K}_2(x) \to \sqrt{\frac{2}{\pi}}\left[2x^{-\frac{3}{2}} + \dots\right]\,.\end{aligned} \tag{2.102}$$

Consider first the computation of the quantities (2.97). As we just mentioned, rather than computing them directly as infinte sums over KK mode masses, we derive them as particular cases of the propagator $\Delta(p,y,y')$ for φ, which is given by the solution with Neumann boundary conditions at y equal to 0 and πR of the following differential equation:

$$\left(e^{-2ky}p^2 - \partial_y e^{-4ky}\partial_y\right)\Delta(p,y,y') = \delta(y-y')\,. \tag{2.103}$$

The solution of this equation is most easily found by switching to the new variable $z = e^{ky}/k$. In these conformal coordinates, the positions of the two branes are given by $z_0 = 1/k$ and $z_1 = e^{k\pi R}/k$ and the metric is

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)\,. \tag{2.104}$$

Notice that after going to conformal coordinates we can treat z_0 as a free parameter playing the role of the conformal compensator at the UV brane. Indeed the change of coordinate $z \to z\lambda$ is equivalent to a shift $z_{0,1} \to \lambda z_{0,1}$ of both boundaries plus a Weyl rescaling $g_{\mu\nu} \to g_{\mu\nu}/\lambda^2$ of the metric along the 4d slices. Therefore both $1/z_0$ and $1/z_1$ have the property of conformal compensator. By locality it is then natural to identify them at the superfield level with the superconformal compensators at the respective boundaries: $1/z_0^2 \to \phi\phi^\dagger$ and $1/z_1^2 = \phi\phi^\dagger e^{-(T+T^\dagger)}$.

Now, defining also $u = \min(z,z')$ and $v = \max(z,z')$, the propagator is [62, 63]:

$$\Delta(p,u,v) = \frac{\left[\hat{I}_1(pz_0)\hat{K}_2(pu) + \hat{K}_1(pz_0)\hat{I}_2(pu)\right]\left[\hat{I}_1(pz_1)\hat{K}_2(pv) + \hat{K}_1(pz_1)\hat{I}_2(pv)\right]}{2p\,(ku)^{-\frac{3}{2}}(kv)^{-\frac{3}{2}}\left[\hat{I}_1(pz_1)\hat{K}_1(pz_0) - \hat{K}_1(pz_1)\hat{I}_1(pz_0)\right]} \tag{2.105}$$

The brane restrictions of the general propagator (2.105) defining eqs. (2.97) are then easily computed. The factors $e^{-\frac{3}{2}ky_{i,j}}$ that have been introduced cancel the factors $kz_{i,j}^{3/2}$ appearing in (2.105). Moreover, one of the factors in the numerator is always trivially equal to 1 thanks to eq. (2.100). The results finally read

$$\Delta_{00}(p) = \frac{1}{2p}\frac{\hat{I}_1(pz_1)\hat{K}_2(pz_0)+\hat{K}_1(pz_1)\hat{I}_2(pz_0)}{\hat{I}_1(pz_1)\hat{K}_1(pz_0)-\hat{K}_1(pz_1)\hat{I}_1(pz_0)}\,, \tag{2.106}$$

$$\Delta_{11}(p) = \frac{1}{2p}\frac{\hat{I}_1(pz_0)\hat{K}_2(pz_1)+\hat{K}_1(pz_0)\hat{I}_2(pz_1)}{\hat{I}_1(pz_1)\hat{K}_1(pz_0)-\hat{K}_1(pz_1)\hat{I}_1(pz_0)}\,, \tag{2.107}$$

$$\Delta_{01,10}(p) = \frac{1}{2p}\frac{1}{\hat{I}_1(pz_1)\hat{K}_1(pz_0)-\hat{K}_1(pz_1)\hat{I}_1(pz_0)}\,. \tag{2.108}$$

It is easy to check that in the limit $k \ll 1/R$ the above propagators correctly reproduce the flat space result. For instance

$$\lim_{kR\to 0}\Delta_{00} = \frac{1}{2p}\coth(\pi pR). \tag{2.109}$$

Since the quantity we want to calculate will contain UV divergent contributions we must first classify these in order to be able to subtract them. The UV divergences are local so they will correspond to renormalizations of the tree level effective action. By inspecting eqs. (2.78,2.85) we the expect the UV divergences to have the general form

$$\Omega_{UV} = \frac{F_0(\Phi_0)}{z_0^2}+\frac{F_1(\Phi_\pi)}{z_1^2}. \tag{2.110}$$

Basically these divergent terms can come from three different sources: the renormalization of the 5d Planck mass and the renormalization of the kinetic funtions at each boundary. Of course throughout this discussion $1/z_0^2$ and $1/z_1^2$ should be thought as the corresponding superfield compensators as discussed above. Moreover, covariance under the Weyl shift $z_{0,1}\to\lambda z_{0,1}$ we just discussed, constrains the Kahler function to have the form

$$\Omega_{1-loop} = \frac{1}{z_0^2}\omega(z_0^2/z_1^2) \equiv t_0\,\omega(t_1/t_0) \tag{2.111}$$

where we have defined convenient variables $t_{0,1} = 1/z_{0,1}^2$. We have also not displayed the dependence on the boundary matter fields, as that is not constrained by Weyl symmetry. By the structure of the UV divergences in eq. (2.110) it follwos that the derivative quantity

$$t_0\partial_{t_0}\partial_{t_1}\Omega_{1-loop} = -\frac{t_1}{t_0}\omega''(t_1/t_0) = -x\frac{d^2\omega(x)}{dx^2} \tag{2.112}$$

must be finite. So one way to proceed is to first calculate ω'' and then reconstruct the full ω by solving an ordinary second order differential equation. This solution is determined up to two integration constants associated to the general solution of the omogeneous equation $\omega''=0$: $\omega=F_0+F_1x$. These constants precisely parametrize, as they should, the UV divergences in eq. (2.110).

In what follows, however, we will not directly apply the above derivative method. We shall instead regulate the loop integral which defines Ω_{1-loop} by adding a suitable function which is manifestly annihilated by the operator $\partial_{t_0}\partial_{t_1}$. The finite result we find then contains all the finite calculable pieces. To define our regulating funtion we study the asymptotic behaviour of the propagators for $p\to\infty$. Up to exponentially suppressed terms of order $e^{-p(z_1-z_0)}$ which are obviously irrelevant we find

$$\lim_{p\to\infty}\Delta_{00} = \frac{1}{2p}\frac{\hat{K}_2(pz_0)}{\hat{K}_1(pz_0)} = \frac{1}{2p}(1+\dots)\equiv\tilde{\Delta}_{00}(p) \tag{2.113}$$

$$\lim_{p\to\infty}\Delta_{11} = \frac{1}{2p}\frac{\hat{I}_2(pz_0)}{\hat{I}_1(pz_0)} = \frac{1}{2p}(1+\dots)\equiv\tilde{\Delta}_{11}(p) \tag{2.114}$$

$$\lim_{p\to\infty}\tilde{\Delta}_{01,10}(p) = 0\,. \tag{2.115}$$

Next, consider the formal determinant (2.98). Although this is not precisely a propagator, still, as it is a function of the spectrum, it can be functionally related to the propagator in eq. (2.105). Indeed, the masses m_n are defined by the positions of the poles $p=im_n$ of (2.105). These are determined by the vanishing of the denominator, that is by the equation

$$F(im_n)=\hat{I}_1(im_nz_1)\hat{K}_1(im_nz_0)-\hat{K}_1(im_nz_1)\hat{I}_1(im_nz_0)=0\,. \tag{2.116}$$

The infinite product in eq. (2.98) is divergent. More precisely, it has the form of a constant divergent prefactor times a finite function of the momentum. In order to compute the latter, we consider the quantity $\partial_p\ln Z(p)=\sum_n 2p/(p^2+m_n^2)$. The infinite sum over the eigenvalues, which are defined by the transcendental equation (2.116), is now convergent and can be computed with standard techniques, exploiting the so-called Sommerfeld–Watson transform. The result is simply given by $\partial_p\ln F(p)$. This implies that $Z(p)=F(p)$, up to the already mentioned and irrelevant infinite overall constant. Omitting the latter, we have therefore

$$Z(p)=\hat{I}_1(pz_1)\hat{K}_1(pz_0)-\hat{K}_1(pz_1)\hat{I}_1(pz_0)\,. \tag{2.117}$$

For $p\to\infty$ the second term is of order $e^{-p(z_1-z_0)}$ with respect to the first. Therefore the quantity that controls the UV divergences in the Casimir energy is

$$\tilde{Z}(p)=\hat{I}_1(pz_1)\hat{K}_1(pz_0)\,. \tag{2.118}$$

Notice that the effective action is proportional to an integral of $\ln Z$. By the above equation we conclude that UV divergences depending on z_0 and z_1 add up, but that there are no mixed terms. This is as expected according to eq. (2.110).

To compute the effective Kähler potential at 1-loop, we can now proceed exactly as in ref. [23]. To summarize, we first calculate the diagram with n insertion of ρ_i, and the other localized kinetic term $\rho_{i'}$ turned off. This sum depends on the brane to brane propagator $\Delta_{ii}(p)$. We then replace this propagator with a propagator dressed with n insertions of $\rho_{i'}$ and again sum over n. We also include a factor of $\log Z(p)$ which gives the contribution to the effective action when all local kinetic terms are turned off.

Since we have already included a factor $(kz_i)^{-\frac{3}{2}}(kz_j)^{-\frac{3}{2}}$ in the definition of Δ_{ij} compared to the standard propagator defined with a pure δ-function source and no induced metric factor, and the interaction localized at z_i in (2.96) involves a factor $(kz_i)^{-2}$, each factor ρ_i will come along with a factor kz_i. The result is then

$$\Delta\Omega_{1-loop} = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\left(\frac{-4}{p^2}\right)\ln\Big\{ Z(p)\Big[\Big(1-kz_0\rho_0 p^2\Delta_{00}(p)\Big)\Big(1-kz_1\rho_1 p^2\Delta_{11}(p)\Big) - kz_0\rho_0 kz_1\rho_1 p^4\Delta_{01}(p)\Delta_{10}(p)\Big]\Big\}\,. \tag{2.119}$$

This integral is divergent, the divergent contribution corresponding, as we anticipated, to a renormalization of local operators. The way we proceed is to subtract the same expression, but with Z and Δ_{ij} replaced by $\tilde{Z}$ and $\tilde{\Delta}_{ij}$

$$\Omega_{\rm div} = \int \frac{-2}{p^2}\frac{d^4p}{(2\pi)^4}\left[\log\tilde{Z}(p) + \sum_i \log\left(1-kz_i\rho_i\rho_i p^2\tilde{\Delta}_{ii}(p)\right)\right] \tag{2.120}$$

This formal expression, being the sum of terms that depend either on z_0 or on z_1 but not on both, vanishes under the action of $\partial_{t_0}\partial_{t_1}$. So it is a fine regulator. It is also strightforward to realize that the subtrated quantity

$$\Delta\Omega_{\rm eff} = \int \frac{d^4p}{(2\pi)^4}\frac{-2}{p^2}\ln\frac{Z(p)}{\tilde{Z}(p)}\frac{\prod_i\Big(1-kz_i\rho_i p^2\Delta_{ii}(p)\Big) - \prod_i\left(kz_i\rho_i p^2\Delta_{ii'}(p)\right)}{\prod_i\Big(1-kz_i\rho_i p^2\tilde{\Delta}_{ii}(p)\Big)}\,, \tag{2.121}$$

is finite. This formula generalizes the flat case result (6.32) of ref. [23] to the warped case.

It is worth spending a few words on the structure of the UV divergences we have subtracted. As $\partial_{t_0}\partial_{t_1}\Omega_{\rm div} = 0$, it follows that, if properly (ie. covariantly) regulated, our subraction have the same form as eq. (2.110). It is instructive how this dependence comes about when working with a hard momentum cut-off. Let us first concentrate on the term proportional to $\ln\tilde{Z}$. Since $\tilde{Z}(p)$ is the product of a function of pz_0 times a function of pz_1, one can split the subtraction in two pieces depending only on z_0 and z_1, and change integration variables respectively to $v = pz_0$ and $u = pz_1$ in the two distinct contributions

$$\frac{1}{z_0^2}\int \frac{-2}{v^2}\frac{d^4v}{(2\pi)^4}\log\hat{K}_1(v) + \frac{1}{z_1^2}\int \frac{-2}{u^2}\frac{d^4u}{(2\pi)^4}\log\hat{I}_1(u) \tag{2.122}$$

These quantities are UV divergent. They have indeed the expected structure provided the cut-off $\Lambda_v L$ and $\Lambda_u L$ of the v and u integrals do not depend on z_0 and z_1. This has an obvious interpretation. The above two distinct contributions must be associated (after an integral over the 5th dimension) with divergences at the two distinct boundaries. Now, the original momentum integration variable p is not the physical coordinate invariant momentum. At each point in the bulk the physical momentum is $p_{phys} = \sqrt{p_\mu p_\nu g^{\mu\nu}} = pz/L$, so that it is truly the variables v and u that parametrize the physical virtual momentum at each boundary. A covariant cut off procedure should bound the physical rather than the comoving momentum. This explains why we get the right result by choosing a fixed cut-off for v, u rather than for p. It is easy to check that this is indeed what happens when the above integrals are regulated through the introduction of 5-dimensional Pauli-Villars fields of mass Λ. In that case it is indeed found $\Lambda_u = \Lambda_v = \Lambda$. By using the asymptotic expansion of K_1 and I_1 at large argument we do find that the divergent part has the form

$$\left(\frac{1}{z_0^2} - \frac{1}{z_1^2}\right)\left\{(\Lambda L)^3 + \Lambda L\right\} + \left(\frac{1}{z_0^2} + \frac{1}{z_1^2}\right)\ln\Lambda \qquad (2.123)$$

This expression is manifestly invariant, as it should, under the exchange of the two boundaries: $z_0 \to z_1, L \to -L$. The first term corresponds to a renormalization $\delta M_5^3 = \Lambda^3 + \Lambda k^2$ of the 5d Planck mass. The last term is associated to boudary kinetic terms proportional to $k^2 \ln\Lambda$.

Consider next the two terms in the sum of the second term of eq. (2.120). Treting ρ_i as small quantities we can expand the logarithm. This lead to infinitely many terms of the type $-\int d^4p/(2\pi)^4(-2/p^2)(kz_i\rho_i p^2\tilde{\Delta}_{ii}(p))^n/n$, with arbitrary positive integer n. Since $\tilde{\Delta}_{ii}(p)$ is actually a function of pz_i, one can rescale p by z_i and factorize a divergent integral, which as before does not anymore explicilty depend on z_1 and z_0. The same arguments as before concerning covariance apply. We will not repeat them. The integral can be evaluated by using the asymptotic expression for large argument of the functions appearing in its integrand and the dimensionless cut-off ΛL. The result for the n-th term then simplifies to $1/(2^{n+2}n(n+2)\pi^2)(\Lambda^{n+2}L^2)\rho_i^n z_i^{-2}$, and we see that it corresponds to a correction to the 4d potentials Ω_i that is proportional to $\Lambda^{n+2}M_5^{-3n}(-3M_i^2 + \Phi_i\Phi_i^\dagger)^n$ and represents a renormalization of the quantities M_i^2, the wave function multiplying the kinetic term $\Phi_i\Phi_i^\dagger$ of the matter fields and the coeffficients of those higher-dimensional local interactions involving up to n powers of $\Phi_i\Phi_i^\dagger$.

Comparing eq. (2.121) with the general expression (2.28), it is possible to extract all the coefficients C_{n_0,n_1}. We will consider in more details the first coefficients with $n_{0,1} = 0, 1$, which control the vacuum energy and the scalar soft masses that are induced by supersymmetry breaking, as functions of the quantities $\alpha_i = M_i^2/M_5^3$ defining the localized kinetic terms for the bulk fields. Taking suitable derivatives of (2.121) with respect to ρ_i and setting these to $-\alpha_i$, we find:

$$C_{0,0} = -2\int \frac{d^4p}{(2\pi)^4} p^{-2}\Big[\ln\mathfrak{Z}(p) - \ln\tilde{\mathfrak{Z}}(p)\Big]\,, \tag{2.124}$$

$$C_{1,0} = \frac{2}{3M_5^3}(kz_0)\int \frac{d^4p}{(2\pi)^4}\Big[\mathfrak{d}_{00}(p) - \tilde{\mathfrak{d}}_{00}(p)\Big]\,, \tag{2.125}$$

$$C_{0,1} = \frac{2}{3M_5^3}(kz_1)\int \frac{d^4p}{(2\pi)^4}\Big[\mathfrak{d}_{11}(p) - \tilde{\mathfrak{d}}_{11}(p)\Big]\,, \tag{2.126}$$

$$C_{1,1} = \frac{2}{9M_5^6}(kz_0)(kz_1)\int \frac{d^4p}{(2\pi)^4}\,p^2\Big[\mathfrak{d}_{01}(p)\mathfrak{d}_{10}(p)\Big]\,, \tag{2.127}$$

in terms of the following, α_i-dressed, versions of the quantities Z and Δ_{ij} (the corresponding large volume quantities being similarly defined out of $\tilde{Z}$ and $\tilde{\Delta}_{ij}$):

$$\mathfrak{Z} = Z\Big[\Big(1+kz_0\alpha_0p^2\Delta_{00}\Big)\Big(1+kz_1\alpha_1p^2\Delta_{11}\Big) - kz_0\alpha_0kz_1\alpha_1p^4\Delta_{01}\Delta_{10}\Big]\,, \tag{2.128}$$

$$\mathfrak{d}_{00} = \frac{\Delta_{00}\Big(1+kz_1\alpha_1p^2\Delta_{11}\Big) - kz_1\alpha_1p^2\Delta_{01}\Delta_{10}}{\Big(1+kz_0\alpha_0p^2\Delta_{00}\Big)\Big(1+kz_1\alpha_1p^2\Delta_{11}\Big) - kz_0\alpha_0kz_1\alpha_1p^4\Delta_{01}\Delta_{10}}\,, \tag{2.129}$$

$$\mathfrak{d}_{11} = \frac{\Delta_{11}\Big(1+kz_0\alpha_0p^2\Delta_{00}\Big) - kz_0\alpha_0p^2\Delta_{01}\Delta_{10}}{\Big(1+kz_0\alpha_0p^2\Delta_{00}\Big)\Big(1+kz_1\alpha_1p^2\Delta_{11}\Big) - kz_0\alpha_0kz_1\alpha_1p^4\Delta_{01}\Delta_{10}}\,, \tag{2.130}$$

$$\mathfrak{d}_{01,10} = \frac{\Delta_{01,10}}{\Big(1+kz_0\alpha_0p^2\Delta_{00}\Big)\Big(1+kz_1\alpha_1p^2\Delta_{11}\Big) - kz_0\alpha_0kz_1\alpha_1p^4\Delta_{01}\Delta_{10}}\,. \tag{2.131}$$

2.4.1 Results in the absense of localized kinetic terms

Let us consider first the case of vanishing localized kinetic terms, that is $\alpha_i = 0$. The first four relevant terms in the correction to the effective Kähler potential are then given by eqs. (2.124)–(2.127) with $\mathfrak{Z} \to Z$ and $\mathfrak{d}_{ij} \to \Delta_{ij}$, and similarly for the tilded quantities defining the subtractions. In this simplest situation, it turns out that the three matter-dependent corrections (2.125)–(2.127) can be understood in a very simple way from the matter-independent correction (2.124). The point is that matter on branes influences the result in the same way as localized kinetic terms given by $\rho_i = \Phi_i\Phi_i^\dagger/(3M_5^3)$, and that these can be treated as infinitesimal since we are interested only in the leading terms, that are at most linear in each of these ρ_i. The spectrum of bulk modes that are responsible for this Casimir effect is then modified by the presence of the localized kinetic terms in a very simple way: the presence of ρ_0 and ρ_1 at the position of the two branes,z_0 and z_1, have the same effect as shifting the positions of these branes from z_0 to $z_0e^{+k\rho_0/2} \simeq z_0 + k\rho_0z_0/2$ and from z_1 to $z_1e^{-k\rho_1/2} \simeq z_1 - k\rho_1z_1/2$. This can be understood as follows. Working with the y coordinate and in 4d Fourier space, and defining for convenience $\eta_0 = 1$ and

$\eta_1 = -1$, the boundary condition for the scalar field φ at the position of the brane $y = y_i$ and in the presence of a localized kinetic term with coefficients ρ_i is given by:

$$\varphi'(y_i) = -\frac{\eta_i \rho_i}{2} e^{2ky_i} p^2 \varphi(y_i) \,. \tag{2.132}$$

At leading order in the parameter ρ_i, this can be rewritten in terms of the shifted position $y_i' = y_i + \eta_i \rho_i/2$ as

$$\varphi'(y_i') = -\frac{\eta_i \rho_i}{2} \left(e^{2ky_i'} p^2 \varphi(y_i') - \varphi''(y_i')\right) \,. \tag{2.133}$$

Using the equation of motion in the bulk, which reads

$$\varphi''(y) - 4k\varphi'(y) - e^{2ky} p^2 \varphi(y) = 0 \,, \tag{2.134}$$

the righ hand side can then be simplified to $2k\eta_i\rho_i\varphi(y_i')$, and one is therefore finally left with the boundary conditions

$$\varphi'(y_i') = 0 \,. \tag{2.135}$$

that is, the boundary condition for a theory without localized kinetic term, but shifted brane positions. More technically, the actual realization of the above relation in the results (2.125)–(2.127) can be easily verified with the help of the following relations[10], which relate the derivatives with respect to the brane positions z_0 and z_1 of the function Z defining the matter-indepenent effect to the brane-to-brane propagators Δ_{ij} entering into the matter-dependent effects:

$$\frac{\partial}{\partial z_0} \ln\left[(kz_0)^{-3/2}(kz_1)^{-3/2} Z(p)\right] = -p^2 \Delta_{00}(p) \,, \tag{2.136}$$

$$\frac{\partial}{\partial z_1} \ln\left[(kz_0)^{-3/2}(kz_1)^{-3/2} Z(p)\right] = +p^2 \Delta_{11}(p) \,, \tag{2.137}$$

$$\frac{\partial}{\partial z_0}\frac{\partial}{\partial z_1} \ln\left[(kz_0)^{-3/2}(kz_1)^{-3/2} Z(p)\right] = -p^4 \Delta_{01}(p)\Delta_{10}(p) \,. \tag{2.138}$$

Using these relations and their analogs for tilded quantities to expand (2.124) around the original positions z_0 and z_1, its is trivial to verify that eqs. (2.125)–(2.127) are indeed correctly reproduced.

In the limit of flat geometry, one recovers the kown results for the coefficients of the four leading operators:

$$C_{0,0} = \frac{c}{4\pi^2} \frac{1}{(T+T^\dagger)^2} \,, \tag{2.139}$$

$$C_{1,0} = \frac{c}{6\pi^2 M_5^3} \frac{1}{(T+T^\dagger)^3} \,, \tag{2.140}$$

$$C_{0,1} = \frac{c}{6\pi^2 M_5^3} \frac{1}{(T+T^\dagger)^3} \,, \tag{2.141}$$

$$C_{1,1} = \frac{c}{6\pi^2 M_5^6} \frac{1}{(T+T^\dagger)^4} \,, \tag{2.142}$$

[10] These relations can be proven by using the fact that $\hat{I}_1'(x) = 3/(2x)\hat{I}_1(x) + \hat{I}_2(x)$ and similarly $\hat{K}_1'(x) = 3/(2x)\hat{K}_1(x) - \hat{K}_2(x)$.

with $c = \zeta(3) = 1.202$. According to the general discussion above, it should be possible to obtain the leading part of the effective Kähler potential corresponding to these first four coefficients from the sole matter-independent term with a suitable shift in the distance between the branes. Since in the flat limit $T + T^\dagger$ is given by $2(z_1 - z_0)$, and the shifted positions are $z_0 + \rho_0/2$ and $z_1 - \rho_1/2$, the shifted distance to be used is $T + T^\dagger - \rho_0 - \rho_1$. Indeed, one can easily verify[11] that eqs. (2.139)–(2.142) are correctly reproduced by expanding at leading order the following expression:

$$\Delta\Omega_{\rm eff} \simeq \frac{c}{4\pi^2}\left[T + T^\dagger - \frac{1}{3}\frac{\Phi_0\Phi_0^\dagger}{M_5^3} - \frac{1}{3}\frac{\Phi_1\Phi_1^\dagger}{M_5^3}\right]^{-2} . \tag{2.143}$$

In the limit of very warped geometry, instead, the coefficients of the first four operators are found to be:

$$C_{0,0} = \frac{ck^2}{4\pi^2}e^{-2k(T+T^\dagger)} , \tag{2.144}$$

$$C_{1,0} = \frac{ck^3}{12\pi^2 M_5^3}e^{-2k(T+T^\dagger)} , \tag{2.145}$$

$$C_{0,1} = \frac{ck^3}{6\pi^2 M_5^3}e^{-2k(T+T^\dagger)} , \tag{2.146}$$

$$C_{1,1} = \frac{ck^4}{18\pi^2 M_5^6}e^{-2k(T+T^\dagger)} , \tag{2.147}$$

where[12]

$$c = \frac{1}{2}\int_0^\infty dx x^3 \frac{K_1(x)}{I_1(x)} = \frac{1}{8}\int_0^\infty dx x^3 \frac{1}{I_1(x)^2} = 1.165 . \tag{2.148}$$

Again, according to our general discussion above, it should be possible to obtain the corresponding leading part of the correction to the Kähler potential by expanding the matter-independent term evaluated with a shifted brane separation. Since in this case $k^2 e^{-2k(T+T^\dagger)}$ is given by $z_0^2 z_1^{-4}$, and the shifted positions are $z_0 e^{+k\rho_0/2}$ and $z_1 e^{-k\rho_1/2}$, the shifted distance to be used is $T + T^\dagger - \rho_0/2 - \rho_1$. Indeed, one can easily verify that eqs. (2.144)–(2.147) are correctly reproduced by expanding at leading order the following expression:

$$\Delta\Omega_{\rm eff} \simeq \frac{ck^2}{4\pi^2}\exp\left\{-2k\left[T + T^\dagger - \frac{1}{6}\frac{\Phi_0\Phi_0^\dagger}{M_5^3} - \frac{1}{3}\frac{\Phi_1\Phi_1^\dagger}{M_5^3}\right]\right\} . \tag{2.149}$$

[11] We thank A. Falkowski for first pointing out this to us as an apparent coincidence.

[12] We were not able to prove that the two integrals coincide, but they numerically agree to a very high accuracy.

2.4.2 Results in the presence of localized kinetic terms

In the presence of localized kinetic terms, that is $\alpha_i \neq 0$, it is convenient to consider directly the full Kähler potential. In the flat case, the result is found to be:

$$\Delta\Omega_{\rm eff} = \frac{1}{4\pi^2}\frac{1}{(T+T^\dagger)^2} f\Big(\frac{\rho_0}{T+T^\dagger}, \frac{\rho_1}{T+T^\dagger}\Big) \tag{2.150}$$

where now $\rho_i = -\alpha_i + \Phi_i\Phi_i^\dagger/(3M_5^3)$ and

$$f(a_0,a_1) = -\int_0^\infty dx\, x \ln\left[1 - \frac{1+a_0 x/2}{1-a_0 x/2}\,\frac{1+a_1 x/2}{1-a_1 x/2}\, e^{-x}\right] . \tag{2.151}$$

Expanding this expression at leading order in the matter fields, one finds[13]

$$C_{n_0,n_1} = \frac{1}{4\cdot 3^{n_0+n_1}\pi^2 M_5^{3n_0+3n_1}}\,\frac{1}{(T+T^\dagger)^{2+n_0+n_1}}\, f^{(n_0,n_1)}\Big(\frac{-\alpha_0}{T+T^\dagger}, \frac{-\alpha_1}{T+T^\dagger}\Big) . \tag{2.152}$$

It is easy to verify that $C_{0,0}$ and $C_{0,1}$ become negative for large α_0 and small α_1, and similarly that $C_{0,0}$ and $C_{1,0}$ become negative for large α_1 and small α_0.

In limit of large warping, one finds instead:

$$\Delta\Omega_{\rm eff} = \frac{k^2}{4\pi^2} e^{-2k(T+T^\dagger)} f(k\rho_0, k\rho_1) \tag{2.153}$$

where $\rho_i = -\alpha_i + \Phi_i\Phi_i^\dagger/(3M_5^3)$ and now

$$f(a_0,a_1) = \frac{1}{2}\int_0^\infty dx\, x^3\, \frac{K_1(x)}{I_1(x)}\,\frac{1}{1-a_0}\,\frac{1+a_1 x/2\, K_2(x)/K_1(x)}{1-a_1 x/2\, I_2(x)/I_1(x)} . \tag{2.154}$$

Expanding this expression at leading order in the matter fields, one deduces

$$C_{n_0,n_1} = \frac{k^{2+n_0+n_1}}{4\cdot 3^{n_0+n_1}\pi^2 M_5^{3n_0+3n_1}} e^{-2k(T+T^\dagger)} f^{(n_0,n_1)}(-\alpha_0 k, -\alpha_1 k) . \tag{2.155}$$

It is easy to verify that none of the coefficients becomes negative for large α_0 and small α_1, whereas $C_{0,0}$ and $C_{1,0}$ become negative for large α_1 and small α_0.

2.5 Conclusions

We computed the 1-loop correction to the Kähler effective potential. The result is encoded in the functions C_{n_0,n_1}, they control the leading effects allowing the trasmission of supersymmetry breaking from one sector to the other. Let us examine the consequences of our results. We will consider a generic situation where supersymmetry breaking occurs through some unknown dynamics and is effectively described through a Goldstone

[13] We use the standard notation $f^{(n_0,n_1)}(a_0,a_1) = (\partial/\partial a_0)^{n_0}(\partial/\partial a_1)^{n_1} f(a_0,a_1)$.

supermultiplet X with a linear superpotential. Similarly, the radion is stabilized by some unspecified dynamics that we shall parametrize through an effective superpotential depending on the radion multiplet. Finally, in order to cancel the cosmological constant, we also need to add a constant superpotential and tune its coefficient. All these superpotentials admit microscopic realizations, for instance in terms of gaugino condensations, but we will not discuss them here in any detail. What is instead important for us is that in such a general situation, there are three important sources of supersymmetry breaking effects for the visible sector matter and gauge fields, coming respectively from the F terms of the compensator, the radion and the Goldstone chiral multiplets S, T and X. These will induce contributions to the soft masses corresponding to anomaly, radion, and brane-to-brane mediation effects.

2.5.1 Flat case

Let us first briefly recall the situation in flat space. We put the visible sector at z_0 and the hidden sector at z_1, but the opposite choice is clearly equivalent. The gaugino masses receive a one-loop contribution from anomaly mediation, but no one-loop contribution from radion and brane-to-brane mediation, whereas the scalar squared masses receive a two-loop contributions from anomaly mediation and a one-loop contribution from radion and brane-to-brane mediation. Their expressions read

$$m_{1/2} = a\Big(\frac{g^2}{16\pi^2}\Big)|F_S| \,, \tag{2.156}$$

$$m_0^2 = b\Big(\frac{g^2}{16\pi^2}\Big)^2|F_S|^2 - C''_{1,0}(T+T^*)|F_T|^2 - C_{1,1}(T+T^*)|F_X|^2 \,. \tag{2.157}$$

The numerical coefficients a and b depend on the quantum numbers of the particular gaugino and scalar partner. The value of the former is qualitatively irrelevant, whereas the latter is positive for squarks but negative for sleptons, leading to a potential problem. The functions $C_{1,0}$ and $C_{1,1}$ have instead been derived in section 2.4.1 and behave respectively like $M_5^{-3}(T+T^*)^{-3}$ and $M_5^{-6}(T+T^*)^{-4}$ times coefficients that are functions of the dimensionless variables $\varepsilon_i = M_i^2 M_5^{-3}(T+T^*)^{-1}$. In the basic situation where $\varepsilon_0 = 0$ and $\varepsilon_1 = 0$, these two coefficients are both positive and of comparable magnitude, leading to negative contributions in eq. (2.157), which worsen the tachyon problem already occurring in the anomaly mediation contribution. For $\varepsilon_0 = 0$ and $\varepsilon_1 \gg 1$, on the other hand, the former becomes negative and remains sizable whereas the latter stays positive but becomes small. Actually, this interesting situation is achieved already for $\varepsilon_0 = 0$ and $\varepsilon_1 \sim 1$, and we shall therefore have in mind these values.

The values for the radion field and the F terms are difficult to derive for general values of the localized kinetic terms. However, in the mostly interesting case where these are sizable but not huge, they will affect the results only in a mild quantitative way, and to understand the qualitative behavior of the result we can therefore neglect them. Assuming that $\Lambda_1 \gg \Lambda_2$, there is a large T solution and, since $F_S \sim F_T/T \sim F_X/M$, the contributions from radion and brane-to-brane mediation have the same magnitude and can compete with the contribution from anomaly mediation if $m_{KK}/M \sim g^2/16\pi^2$, where $m_{KK} = 1/R$. In this situation, we then get rid of any flavor or tachyon problem and obtain:

$$m_0 \sim m_{1/2} \sim \left(\tfrac{g^2}{16\pi^2}\right) m_{3/2} \,, \tag{2.158}$$

$$m_{\rm scalar} \sim m_{\rm pseudoscalar} \sim m_{3/2} \,. \tag{2.159}$$

2.5.2 Warped case

In the warped case, the situation is similar, but the two possible choices for the locations of the visible and hidden sectors are no longer equivalent and must be studied separately. It will be convenient to use the variables S and $\omega = Se^{-kT}$ instead of S and T. These are indeed the fields that effectively act as conformal compensators at the UV and IR branes respectively.

Let us consider first the scenario where the visible sector is on the UV brane at z_0 and the hidden sector on the IR brane at z_1. Since the matter and gauge fields live on the UV brane, they have a canonically normalized kinetic term and they couple to the ordinary conformal compensator S. The soft masses are then given by

$$m_{1/2} = a\Big(\frac{g^2}{16\pi^2}\Big)\,|F_S| \,, \tag{2.160}$$

$$m_0^2 = b\left(\frac{g^2}{16\pi^2}\right)^2 |F_S|^2 - C_{1,0}''(|\omega|)|F_\omega|^2 - C_{1,1}(|\omega|)|F_X|^2 \,. \tag{2.161}$$

The numerical coefficients a and b are the same as before, but the functions $C_{1,0}$ and $C_{1,1}$ are now different. They have been derived in section 2.4.2 and in the limit of large warping they behave respectively like $k^3M_5^{-3}|\omega|^4$ and $k^4M_5^{-6}|\omega|^4$, times coefficients that are functions of the new dimensionless variables $\varepsilon_i = kM_i^2M_5^{-3}$. As in the flat case, these coefficients are both positive for $\varepsilon_0 = 0$ and $\varepsilon_1 = 0$, but for $\varepsilon_0 = 0$ and $\varepsilon_1 \gg 1$ the first becomes negative and remains sizable, whereas the second remains positive but becomes small. We therefore have the same potentially interesting case as in a flat extradimension for ε_0 and $\varepsilon_1 \sim 1$.

Let us consider next the scenario where the visible sector is on the IR brane at z_1 and the hidden sector on the UV brane at z_0. Since the matter fields live now on the IR brane, they have a non-canonical kinetic term involving an $|\omega|^2$ factor. The gauge fields, instead, still have a canonical kinetic term, because their action is conformal and insensitive to the induced metric. Moreover, both the matter and the gauge fields couple to the red-shifted conformal compensator ω. The physical soft masses are then given by

$$m_{1/2} = a\Big(\frac{g^2}{16\pi^2}\Big)\,|F_\omega| \,, \tag{2.162}$$

$$m_0^2 = \left[b\left(\frac{g^2}{16\pi^2}\right)^2 |F_\omega|^2 - C_{0,1}''(|\omega|)|F_\omega|^2 - C_{1,1}(|\omega|)|F_X|^2\right]|\omega|^{-2} \,. \tag{2.163}$$

The function $C_{0,1}$ has the same behavior as the function $C_{1,0}$ that was relevant for the previous case, namely $k^3 M_5^{-3}|\omega|^4$, but its coefficient can never change sign, even for $\varepsilon_0 \gg 1$ and $\varepsilon_1 = 0$. In fact, the two coefficients of the functions $C_{0,1}$ and $C_{1,1}$ stay both positive for any value of ε_0 and ε_1, whose effect is only to decrease their size. No interesting situation can therefore emerge from this case and we will not study it further.

In the warped case, there are in principle two inequivalent ways of realizing the general situation described at the beginning of the section for supersymmetry breaking and radion stabilization. Indeed, the constant superpotential that has to be added, in order to tune the cosmological constant to zero, can be located at any of the two branes, but the two choices are not equivalent, because of the different conformal compensators that are active at the two different branes. We focus on the interesting scenario where the visible sector is at the UV brane and the hidden at the IR brane. As before, the values for the radion field and the F terms are difficult to derive for general values of the localized kinetic terms, but in order to get insight on the qualitative behavior we can neglect the latter.

If the constant superpotential entirely comes from the visible sector we then get $F_S \sim F_\omega/\omega \sim \omega F_X/M$. This implies that the contribution from radion mediation is parametrically smaller than the others, and cannot help to change the sign of the scalar squared masses.

In the case in which the constant superpotential instead comes from the hidden sector we have $F_S \sim F_\omega \sim \omega F_X/M$. This implies that the contributions to m_0^2 in eq. (2.161) from radion and brane-to-brane mediations have the same magnitude and can compete with the contribution from anomaly mediation if $m_{\rm KK}/M \sim g^2/16\pi^2$, where $m_{\rm KK} = k|\omega|$. In this situation, we get again rid of the flavour and tachyon problems, and obtain:

$$m_0 \sim m_{1/2} \sim \left(\tfrac{g^2}{16\pi^2}\right) m_{3/2} \;, \tag{2.164}$$

$$m_{\rm scalar} \sim \omega^{-1} m_{3/2} \;, \quad m_{\rm pseudoscalar} \sim \omega^{-1/2} m_{3/2} \;. \tag{2.165}$$

The situation is therefore very similar to the one emerging in the flat case, the main difference being an enhancement of the moduli masses with respect to the soft masses.

CHAPTER 3

Gauge couplings unification

3.1 Holographic interpretation of the running

In this chapter, we consider GUT models where the gauge bosons of the unified group propagate in the AdS bulk and study how this reflects on the low energy gauge couplings. For a description of this scenario and its holographic interpretation see chapter 1.

At energies much greater than the TeV scale, the KK states become strongly coupled. Nevertheless, if we restrict to the study of inclusive quantities, given by Green functions on the Planck brane, we can reach energies as high as the Planck scale without entering a strong coupling regime [39, 40] (see also [64]). This is possible because of the exponential die-off of the propagators in the bulk, $G \sim e^{-\sqrt{p^2}z}$ at distances $z \gtrsim p^{-1}$, which makes the high energy processes on the Planck brane insensible of what is going on deep inside AdS: the local cut-off for an observer living on the Planck brane is given by the AdS curvature k. The importance of these inclusive quantities is clear also from the holographic point of view, having the brane-brane correlators a simple 4-dimensional meaning. In our case, the gauge propagator between two points on the Planck brane tells us the strength of the gauge interaction in the 4-dimensional dual theory and it remains perturbative despite the fact that the KK gauge bosons become strongly coupled above the TeV. This dual picture allows us to understand why the gauge coupling running is still logarithmic above the TeV scale: the CFT composites become broader and broader and the true degrees of freedom emerge, but their contribution to the running still remain perturbative and 4-dimensional, *i.e.* logarithmic. In a unified model, brane-brane gauge correlators for different groups are the same much above the unification scale and this may happen in a regime ($E \gg$ TeV) in which only boundary correlators make sense.

At energies much greater than the TeV scale, but smaller than the AdS curvature k, the tree level Planck brane-brane gauge propagator is given by [29]

$$G(q) = \frac{g_5^2/L}{q^2(\log(2k/q) - \gamma)} , \tag{3.1}$$

where γ is the Euler-Mascheroni constant. The holographic interpretation of this formula is quite simple [39]. It just describes the corrections to a 4d vector propagator

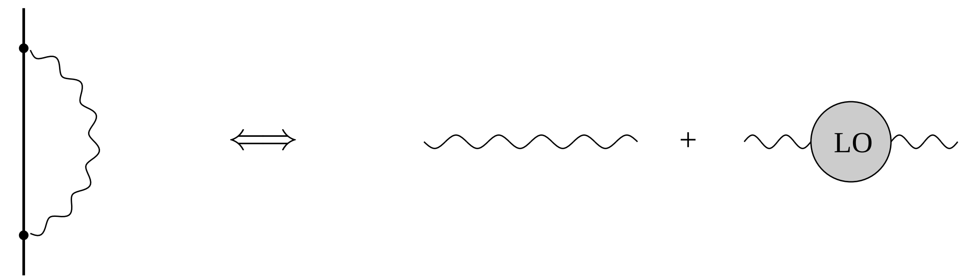


Figure 3.1: The brane-brane correlator in AdS corresponds holographically to the free gauge propagator corrected by the LO contribution in $1/N$ of the CFT (of order $\sim \mathcal{O}[N^2(\alpha/4\pi)]$ with respect to the tree level). The grey circle represents the $\langle JJ \rangle$ insertion.

given by $\langle JJ \rangle_{\mathrm{CFT}}$ insertions, where J is the CFT current coupled to the gauge boson (see figure 3.1). Conformal invariance tells us that $\langle J(p)J(-p)\rangle \propto p^2 \log p^2$, so that the logarithmic running (3.1) follows. It is worthwhile noting that this CFT running is simply described by the tree level AdS propagator and it is common to any gauge group. It follows that, whatever GUT symmetry breaking mechanism we choose, *the leading CFT contribution to the running is always GUT invariant*. From eq. (3.1) we see that the CFT gives a positive contribution to the beta-function $b_{\mathrm{CFT}} = 8\pi^2 L/g_5^2 \sim N^2$, where N is the number of colors of the conformal theory [65]. This should be large to ensure that the non-renormalizable 5d gauge theory makes sense: the AdS curvature k must be much smaller than the cut-off scale $\Lambda = 24\pi^3/g_5^2$ or, equivalently, the number N of colors should be large.

3.1.1 Radiative corrections to brane correlators

Additional contributions to the running of the gauge couplings come from loop corrections to the brane-brane propagator. It is therefore natural to ask what is the holographic interpretation of these loops. For example, what is the 4-dimensional counterpart of the vacuum polarization due to a bulk scalar? In the limit in which we remove the Planck brane, obtaining a complete AdS space, we know that the dual picture is simply a CFT. In this case, bulk loops are interpreted as corrections to the CFT correlators, subleading in a $1/N$ expansion [66]. Concerning the scalar loop correction to the gauge two-point function, we would find a modification of the $\langle JJ \rangle$ CFT correlator. As the dependence of this correlator on the 4d momentum is fixed by conformal invariance, only the coefficient in front receives $1/N$ corrections.

What changes if we add the Planck brane? The rough picture is the following. Cutting off the part of AdS space near its boundary corresponds to a UV modification of the CFT, which is now smeared over a distance of order k^{-1}: degrees of freedom of shorter wavelength have been integrated out. Moreover the 4d role of fields living in AdS space changes. In the full AdS case they are not dynamical from the 4d point of view: their

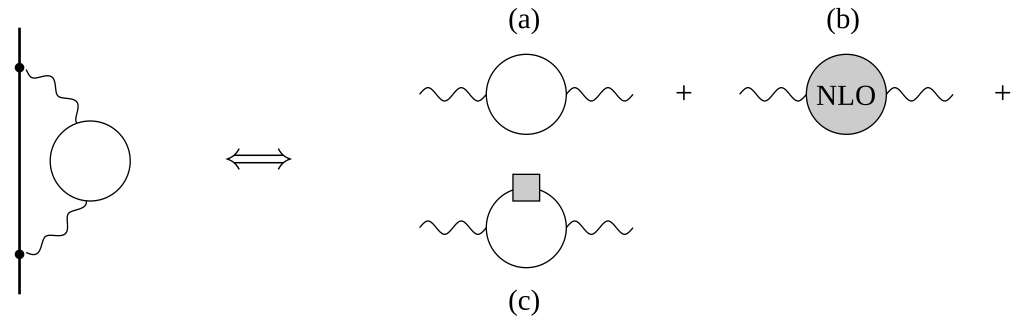


Figure 3.2: The one-loop (rainbow) scalar correction to the brane-brane correlator in AdS corresponds holographically to three different diagrams: a 4d scalar loop graph (a), the same diagram with the scalar propagator corrected by the CFT (c), and the NLO contribution in $1/N$ of the CFT (b). Diagrams (a), (b) are both $\mathscr{O}(\alpha/4\pi)$ with respect to the tree level; diagram (c) is negligible because the scalar coupling to the CFT is M_{Pl}-suppressed. The grey circle (square) represents the $\langle JJ\rangle$ ($\langle OO\rangle$) insertion. A similar holographic interpretation holds for the seagull diagram.

boundary behaviour at infinity just acts as a source for the corresponding operator of the CFT. With the addition of the Planck brane, bulk fields become dynamical also from the 4d viewpoint, as we must integrate over their boundary value on the brane.

We thus expect that radiative corrections to brane correlators in presence of the Planck brane describe not only $1/N$ subleading CFT terms, but the additional contribution of the 4d fields made dynamical by the introduction of the brane. If we have a scalar field in AdS cut by the Planck brane, the 4d theory contains a dynamical scalar, coupled to the CFT through an operator $O(x)$, which has dimension 4 if the scalar is massless. Loops of this 4d scalar will enter the running of the gauge couplings.

As depicted in figure 3.2, the one-loop AdS correction corresponds to the sum of different terms: the contribution from the 4d scalar (a), whose propagator gets itself a CFT correction (c), and the NLO CFT insertion (b). It is worth noting that the various terms can be arranged in a double expansion: the first is the standard series in powers of $(\alpha/4\pi)$, the second is the expansion of the CFT correlators in powers of $1/N$. The two expansions are related, as the holographic prescription tells us that $1/N^2 \sim g_5^2/16\pi^2 L$. Diagrams (a) and (b) are of order $\mathscr{O}(\alpha/4\pi)$ with respect to the tree level; diagram (c) is completely negligible in this case, being the CFT coupled to the 4d scalar only through M_{Pl}-suppressed operators. The corresponding diagram in the case of vector boson loops is $\mathscr{O}[N^2(\alpha/4\pi)^2]$, but still subleading with respect to the other two contributions, as 4d perturbativity requires $N^2(\alpha/4\pi) \ll 1$.

We can look at the contribution (b) and (a) in fig. 3.2 as coming respectively from the limiting case of a 5d loop deep inside AdS or close to the Planck brane. This is quite intuitive, as the 4d scalar field comes from the integration over the boundary conditions on the Planck brane. In the complete AdS case, the boundary values $\phi_0, A^0_\mu, g^0_{\mu\nu}$ for the various fields at infinity act as sources for the corresponding operators in the CFT [4]:

$$\langle e^{-\int d^4x\, O(x)\phi^0(x)+J^\mu(x)A^0_\mu(x)+T^{\mu\nu}(x)g^0_{\mu\nu}(x)}\rangle_{\rm CFT}=e^{-S_{\rm AdS}(\phi^0,A^0_\mu,g^0_{\mu\nu})}\,. \tag{3.2}$$

The right hand side of this equation must be regularized [4], and this procedure leads us closer to the truncated AdS case we are interested in. The standard procedure is to limit the z integration to $z>\varepsilon$ (which corresponds to introducing an explicit UV cut-off on the CFT), add a proper local counterterm action (divergent for $\varepsilon\to 0$) function of $\phi^0,A^0_\mu,g^0_{\mu\nu}$ and their derivatives, and then take the limit $\varepsilon\to 0$. In the case with only a scalar field, eq. (3.2) becomes

$$\langle e^{-\int d^4x\, O(x)\phi^0(x)}\rangle_{\rm CFT}=\lim_{\varepsilon\to 0} e^{-S_{\rm AdS}(\phi^0,\varepsilon)}e^{-S_{\rm count}(\phi^0,\varepsilon)}\,. \tag{3.3}$$

Suppose now not to perform the final limit, keeping an explicitly truncated AdS space. As we have integrated out a portion of space which corresponds to the UV of the CFT, we expect this to correspond to a smearing procedure in which fast modes are integrated out [1] [37, 38, 67, 68]. At this stage, the scalar loop correction in AdS of figure 3.2 gives a subleading contribution to the $\langle JJ\rangle_{\phi_0}$ CFT correlator in the external background ϕ_0.

The last step to get the Randall-Sundrum scenario is to integrate over the boundary values $\phi^0,A^0_\mu,g^0_{\mu\nu}$, which become dynamical fields, introducing a generic brane action $S_{\rm bound}(\phi_0)$. Consider for instance a brane action with only a kinetic term proportional to an arbitrary parameter ξ:

$$S_{\rm bound}(\phi_0)=\frac{\xi}{k}\int_{\rm brane} d^4x\sqrt{g}\,\partial_\mu\phi\,\partial_\nu\phi^\dagger g^{\mu\nu}\,. \tag{3.4}$$

By varying ξ one changes the kinetic term of the 4d scalar and therefore the relative importance between its loop contribution (fig. 3.2a) and the CFT correction (fig. 3.2b) [2]. In the limit $\xi\to+\infty$, the 4d scalar is frozen out and we are left with the CFT correction; the same result holds by choosing Dirichlet boundary conditions on the Planck brane. From these considerations, it should be clear the strict connection between boundary terms in AdS and the 4d scalar mode. Moreover, all the features of the AdS bulk reflect on the CFT. In particular, if the GUT symmetry is unbroken in the bulk, the CFT is GUT-preserving at all orders.

[1] The counterterm action contains an infinite series of increasing dimension, properly suppressed by powers of k: at energies of order of the AdS curvature the theory becomes non-local. In the following we concentrate on the lower dimension operator $(\partial\phi_0)^2$.

[2] Note however that, if the integration over the boundary conditions were done after the addition of the counterterm $S_{\rm count}(\phi_0)$ (eq. (3.3)), the theory would be ill-defined. This is expected because, in this case, the dependence of the 4d action on ϕ_0 is given by the two point function $\langle OO\rangle_{\rm CFT}$ of the corresponding operator [4]: $S(\phi^0)\propto\int d^4x d^4x'\phi^0(x)\phi^0(x')/|x-x'|^8$. This implies that, if the field ϕ_0 is seen as dynamical, it has a non-local kinetic term of the form $q^4\log q$. This property can be used to find $S_{\rm count}(\phi_0)$ at leading order. To fix it we *require* that the 5d brane-brane propagator goes as $1/(q^4\log q)$. It is easy to obtain that this happens for $\xi=-1$. For $\xi>-1$ the integration over the boundary conditions is well defined as the 4d scalar has a conventional kinetic term, with the correct sign.

3.1.2 The CFT contributions

We now concentrate on the pure CFT corrections, as if we had pushed the Planck brane to infinity, recovering the complete AdS space. We have shown that at leading order the $\langle JJ\rangle$ correlator does not distinguish among the unbroken subgroups of a unified theory, while at NLO the CFT correction is GUT invariant or not depending on the mechanism we choose to break the GUT symmetry. If the symmetry is broken by the boundary conditions, the AdS bulk remains GUT invariant as well as the dual CFT [3]. In this case, at subleading order the CFT still gives a common running to all the unbroken subgroups.

Another possibility is that the unified theory is broken in the bulk, through a vev of a charged scalar Σ. If the expectation value of the scalar is constant along the fifth dimension, the conformal symmetry is still unbroken (all AdS isometries are preserved) but the GUT symmetry is not [4]. In the holographic theory, we have turned on an operator O_Σ coupled to the 4d Σ scalar, transforming under the GUT symmetry, which therefore results spontaneously broken [5]. In this scenario, GUT-breaking corrections to Planck brane propagators correspond to analytic or non-analytic operators, involving Σ, in the 5d effective action. The contribution due to analytic operators is not calculable, and can be only estimated through a naive dimensional analysis. For instance, in the case of the ΣFF operator, naive dimensional analysis gives a ratio between the $\mathscr{O}(1)$ and $\mathscr{O}(N^2)$ corrections to the CFT beta-function:

$$\frac{b_{\rm CFT}^{\rm NLO}}{b_{\rm CFT}^{\rm LO}} \sim \frac{g_5\langle\Sigma\rangle}{\Lambda} = \frac{M_{\rm GUT}}{\Lambda}\ , \tag{3.5}$$

where $\Lambda = 24\pi^3/g_5^2$ is the 5d cut-off. The second equality relates this ratio to the mass of the 4d GUT gauge bosons: $M_{\rm GUT} = g_5\langle\Sigma\rangle$. If we want unification to occur in the regime in which the holographic dual makes sense, we have to require $M_{\rm GUT} \ll k$, so that the ratio (3.5) must be quite small.

From the 4d point of view these corrections to the $\langle JJ\rangle_{\rm CFT}$ correlator are due to the multiple Green functions involving the additional operator O_Σ:

$$\langle O_\Sigma O_\Sigma \ldots JJ\rangle_{\rm CFT}\ . \tag{3.6}$$

For example the ΣFF AdS vertex gives at tree level a CFT correlator $\langle O_\Sigma JJ\rangle_{\rm CFT}$ [65]; turning on the O_Σ operator thus modifies the current-current correlators in a non GUT-invariant way. All these corrections are suppressed by powers of $1/N$ and $\lambda \equiv \langle\Sigma\rangle/k^{3/2}$, where λ is the coupling constant of the operator O_Σ in the 4d picture.

[3] If the boundary conditions on the TeV brane break the GUT symmetry, in the dual picture we have a spontaneous breaking of the symmetry at the TeV scale, together with the conformal breakdown. Still, the running above the TeV scale remains GUT invariant, as we will explicitly check in the following sections.

[4] We may also consider operators which break the conformal symmetry, corresponding to massive scalars. In this case the scalar profi le will not be constant in AdS.

[5] In presence of the Planck brane, the GUT symmetry breaking is spontaneous, due to the vacuum expectation value of the Σ fi eld. In the complete AdS case one just turns on the corresponding operator O_Σ, causing an explicit breaking.

Notice that the non-calculability due to higher-dimension operators in AdS is here reflected into an incalculable CFT beta-function. Nevertheless predictability is retained if we assume that the AdS picture is weakly coupled so that the perturbative expansion makes sense, with higher dimension operators properly suppressed by powers of the 5d cut-off Λ. In this case, CFT contributions which distinguish among the unbroken subgroups are suppressed with respect to the GUT invariant leading CFT running. In turn, this leading contribution cannot be too large if we want to build a phenomenologically viable model.

The reason for this is quite simple and it is a general problem of unification in this kind of models: the GUT-invariant CFT running would imply, for $N \gg 1$, that *at low energy we should see nearly SU(5)-invariant couplings:* $\Delta\alpha/\alpha \ll 1$. Alternatively: if N is too large, we meet the strong coupling regime before reaching the unification scale. It is easy to deduce a limit on N from the requirement of perturbativity at the GUT scale (which we take to be of the order of the standard one $M_{\rm GUT} \sim 10^{16}$GeV): $N^2(\alpha_{\rm GUT}/4\pi) \ll 1$, where the N^2 factor comes from the number of CFT states. From this we obtain:

$$b_{\rm CFT} \ll \frac{2\pi\alpha_i^{-1}(\mathrm{TeV}) - b_i^0 \log M_{\rm GUT}/\mathrm{TeV}}{1+\log M_{\rm GUT}/\mathrm{TeV}} \sim 8\,, \tag{3.7}$$

where the numerical bound is obtained for b_i^0 given by the SM matter content. An opposite bound on $b_{\rm CFT}$, or equivalently on N, comes from the requirement of perturbativity in 5d, namely $\Lambda/k\pi \gg 1$, where Λ is the 5d cutoff: the inequality

$$b_{\rm CFT} = \frac{8\pi^2 L}{g_5^2} \gg \frac{1}{3} \tag{3.8}$$

follows.

The general conclusion is that the leading CFT running cannot be much greater than other contributions which separate the unbroken subgroups, coming from additional particles coupled to the CFT. The limit on N is not so strong to spoil the perturbativity of the AdS picture, as we see comparing eqs. (3.7) and (3.8), even if the allowed window is not too wide. This limit on the CFT leading contribution implies, in turn, that subleading corrections, coming from bulk loops and higher dimension operators are negligible with respect to the non-CFT running.

In principle we could discuss the gauge coupling running even in absence of a unified group in the bulk and check if the gauge couplings cross at a certain energy. In this case the CFT contribution to the running of each group is different at leading order, so that the running may be much faster with a consequent lowering of the unification scale [63]. However, the CFT beta-function for the three groups, given at leading order by the three independent gauge kinetic terms, is incalculable, so that no firm prediction seems possible.

Before moving to the explicit calculations, we want to stress an important conceptual difference between the standard models of unification and the ones built in AdS space. In the standard case, Weinberg's approach of effective gauge theory is very useful and it tells us that the details of GUT-symmetry breaking, resulting only in threshold corrections, are not crucial to test unification. Here the situation is different. Modifying the unified gauge group we are at the same time changing the CFT excitations, hopefully around the corner, at the TeV scale. The pattern of symmetry breaking does not influence only the physics at far-away energies, but also the subleading CFT corrections to the running down to the TeV scale. All this follows from the fact that AdS space describes at the same time the CFT properties and the behaviour of the additional particles coupled to it.

3.2 The low energy gauge coupling

In this section we present our result for the one-loop scalar correction to the low energy coupling of a $U(1)$ gauge group in the bulk. In [63] the one-loop correction to the low energy coupling was computed in RSI for a non-abelian gauge theory, employing a momentum cutoff which depends on the fifth dimension. In [64], the case of massless scalar QED was considered, while in [69] also the massive case has been studied, adopting a Pauli-Villars regulator.

In order to regulate the loop divergence we choose the dimensional regularization, which proved to be a powerful scheme also in theories with flat extra-dimensions [70]. In the specific case of the one-loop correction to the zero-mode gauge correlator, it is enough to extend the brane dimension to a generic (complex) value d keeping just one extra dimension. Analogously to the Minkowski case, the isometries of AdS space are clearly preserved. We leave to the appendix B all the computational details, focusing our attention on the holographic interpretation. Once given the main formulae for different boundary conditions of the scalar field, we will able in the next section to discuss various scenarios of GUT symmetry breaking.

The zero-mode gauge self-energy reads, for external 4d momentum p:

$$\frac{1}{g^2(p^2)} = \frac{\log(z_1/z_0)}{kg_5^2} + \Delta_0(\mu) + \Delta_1(\mu) - \Pi(p^2,\mu)\,, \tag{3.9}$$

where μ is the subtraction point and $\Delta_{0,1}(\mu)$ are the coefficients of the gauge kinetic terms localized on the branes. $\Pi(p^2,\mu)$ is the one-loop scalar correction

$$\Pi(p^2,\mu) = -\mu^{4-d} \sum_{\{x_n\}} \int_0^1 dx\, (2x-1)^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{[q^2 + x_n^2 + c^2(x)]^2}\,. \tag{3.10}$$

Here $c^2(x) = x(1-x)(-p^2)$ and x_n is the mass of the n-th Kaluza-Klein mode of the scalar field (see appendix B). Using the technique described in this appendix, it is easy to perform the integration first and then the sum, getting

$$\begin{aligned}\Pi(p^2,\mu) = \frac{(b_0/2)}{8\pi^2}\Bigg[&-\frac{\alpha}{\varepsilon} + \log\left(\sqrt{-p^2}\sqrt{z_0 z_1}\right) + \alpha \log\frac{\sqrt{-p^2}}{\mu} \\ &+ 3\int_0^1 dy\, y\sqrt{1-y^2}\,\log f\left(iy\sqrt{-p^2}/2\right) \\ &+ \alpha\frac{\gamma}{2} + \log\pi - \frac{4}{3}(1+\alpha)\Bigg] .\end{aligned} \tag{3.11}$$

With $b_0 = 1/3$ we mean the beta-function of a charged 4d scalar, and $d = 4-\varepsilon$. The previous formula is a completely general result, valid for a scalar with arbitrary boundary conditions and mass; in the case of $(\pm\pm)$, $(\pm\mp)$ boundary conditions, one should read $\alpha = \pm 1$, $\alpha = 0$ respectively and choose a function $f = f_{\pm\pm}$, $f = f_{\pm\mp}$, whose expression is given in appendix B. In the particular case of a $(++)$ massless scalar, eq. (3.11) coincides with the result of [64].

The zero-mode gauge propagator is an exclusive observable and does not make sense above the TeV where the 0 mode becomes strongly coupled. This means that eq. (3.11) can be really trusted only for external momenta $|p| \lesssim$ TeV [64]; at these energies it matches the Planck brane-brane correlator, therefore admitting a simple holographic interpretation. Once the function f in eq. (3.11) is expanded for $z_1|p| \ll 1$, the logarithmic dependence on the momentum p must be the correct one for an infrared log. The logarithmic divergence, represented by the $1/\varepsilon$ pole, is the same as in the flat limit (for the latter, see [71]). This was expected, because in the very high energy regime the curvature can be neglected and AdS appears locally flat [64, 69].

In the following we collect the low energy limit $z_1|p| \ll 1$ expression of $\Pi(p^2,\mu^2)$ for all possible choices of boundary conditions in the massless case and for a (++) scalar with AdS bulk mass m. Using the asymptotic expansions of eqs. (B.12), we obtain (subtracting the $1/\varepsilon$ divergence and omitting irrelevant constants):

MASSLESS SCALAR $(1/z_1 \gg |p| > z_0/z_1^2)$

$$\Pi_{++}(p^2,\mu) \simeq \frac{b_0}{8\pi^2}\left[\log\frac{z_1}{z_0} + \log z_0\sqrt{-p^2} - \frac{1}{4}\log\mu z_0 - \frac{1}{4}\log\mu z_1\right] \tag{3.12}$$

$$\Pi_{--}(p^2,\mu) \simeq \frac{b_0}{8\pi^2}\left[\log\frac{z_1}{z_0} + \frac{1}{4}\log\mu z_0 + \frac{1}{4}\log\mu z_1\right] \tag{3.13}$$

$$\Pi_{-+}(p^2,\mu) \simeq \frac{b_0}{8\pi^2}\frac{3}{4}\log\frac{z_1}{z_0} \tag{3.14}$$

$$\Pi_{+-}(p^2,\mu) \simeq \frac{b_0}{8\pi^2}\left[\frac{5}{4}\log\frac{z_1}{z_0} + \log z_0\sqrt{-p^2}\right] . \tag{3.15}$$

MASSIVE (++) SCALAR $(k \gg m \gg |p|,\, |p| \ll 1/z_1)$

$$\Pi(p^2,\mu) \simeq \frac{b_0}{8\pi^2}\left[\log\frac{z_1}{z_0} + \log mz_0 - \frac{1}{4}\log\mu z_0 - \frac{1}{4}\log\mu z_1 + \frac{m^2 z_0^2}{8}\left(\log\frac{z_1}{z_0} - \frac{1}{2}\right)\right]. \tag{3.16}$$

MASSIVE (++) SCALAR $(m \gg k \gg 1/z_1 \gg |p|)$

$$\Pi(p^2,\mu) \simeq \frac{b_0}{8\pi^2}\left[\frac{3}{4}\log\frac{z_1}{z_0} + \frac{1}{2}\log\frac{m}{\mu} + \frac{1}{2}z_0\sqrt{m^2}\log\frac{z_1}{z_0}\right]. \tag{3.17}$$

For the $(++)$ scalar these equations agree with the results of [69] if $\mu = k$.

The $\log p$ terms in the previous formulae are the expected infrared logarithms. Holographically they correspond, in the $(++)$ massless case, to the 4d massless mode which runs logarithmically from high scale down to low energy. It can be interpreted as the Goldstone boson of the symmetry $\phi \to \phi + \text{const.}$, which shifts the 5d scalar field by a constant. This symmetry is broken in the $(+-)$ case by the boundary condition on the TeV brane and the Goldstone boson acquires a tiny mass $M \sim z_0/z_1^2 \sim 10^{-4}\,\text{eV}$. That M should be so small can be understood by the following argument: the 4d scalar couples to the CFT with a M_{Pl} suppressed operator and the analog of the pion decay constant is $f_\pi \sim k$. A Dirichlet boundary condition on the TeV brane implies the breaking of the $\phi \to \phi + \text{const.}$ symmetry with a typical breaking scale $\sim$ TeV. Then a mass follows for the pseudo-Goldstone boson $M^2 \sim \text{TeV}^4/f_\pi^2 \sim \text{TeV}^4/k^2$ (the exact value of the mass can be derived as the lightest eigenvalue of the spectrum equation (B.2) given in appendix B [6]). We conclude that, for scalar boundary conditions $(+-)$, the holographic theory contains an almost massless 4d scalar contributing to the running of the gauge coupling down to very low energy [7]. This explains the $\log p$ term in Π_{+-}.

Non-local operators in the bulk corresponds in $\Pi(p^2,\mu)$ to calculable terms. These effects are interpreted holographically as a calculable correction to the CFT beta-function. There is also an additional incalculable correction coming from the linear divergence of the bulk gauge kinetic term, but of course this does not appear in dimensional regularization. Assuming an holographic point of view, one can extract this NLO CFT contribution from any of the equations (3.12)-(3.15). Consider for example eq. (3.12): the $\log pz_0$ term is the running of the 4d scalar from the Planck scale down to p; the latter two terms, coming from the log divergence on the AdS boundary, are threshold corrections in the

[6] Also ref. [72] has recently pointed out the appearance of such a small eigenvalue in the similar case of a boundary mass term on the TeV brane for the scalar field.

[7] In the case of a vector field, Dirichlet boundary conditions on the TeV brane implies a mass $\sim$ TeV. This is expected, because its coupling with the CFT is dimensionless so that the mass is only logarithmically suppressed by $1/\log(k/\text{TeV})$. For a fermion field we obtain $m^2 \sim \text{TeV}^2 \cdot \text{TeV}/k$.

4d theory at the scales $1/z_0$, $1/z_1$. The remaining contribution, namely the first term in eq. (3.12), is the calculable part of the NLO CFT correction. On the other hand, any of the equations (3.12)-(3.15) does not constitutes by itself an unambiguous test of the holographic interpretation. Such an ambiguity can be resolved only by comparing the different results for the various parities as we will do in the next section when we consider the GUT breaking scenarios. In the massive case, eqs. (3.16),(3.17), there are also contributions of the form $z_0^2 m^2 \log z_1/z_0$, $z_0\sqrt{m^2}\log z_1/z_0$; both are calculable, as they correspond to AdS bulk operators which depend non-analytically on the scalar curvature $\mathscr{R}$ (the former) or on the Lagrangian parameter m^2 (the latter). The $z_0\sqrt{m^2}\log z_1/z_0$ terms appear uniquely for a very large value of the mass, $m \gg k$ (see eq. (3.17)). In this limit they represent the contribution of CFT operators of very high dimension $\propto m/k$, while $\log m$ terms can be interpreted only from a 5d point of view: their coefficient $b_0/2$ comes from the running of boundary operators.

It is well known [73] that, in the 5d flat case, gauge kinetic terms on the boundary evolve logarithmically with energy, and their beta-function gets a one-loop contribution from particles living in the bulk. This evolution is intimately connected with a logarithmic divergence. The whole tower of massive KK states contributes to the running on a given boundary with $1/4$ of the beta-function b_0 of the zero-mode, the sign of the effect depending on the parity of the loop fields [71]: including also the zero-mode contribution, one finds $\pm 1/4\, b_0$ if the loop field is $\pm$ on that specific boundary. The logarithmic divergences, together with the associated $\log p/\mu$ terms, cancel in the one-loop correction from a $(\pm,\mp)$ scalar to the zero-mode gauge propagator, summing the contributions at the two boundaries. All these considerations must remain valid in the warped case as well, being the divergences the same as in the flat limit. This can be verified looking at eqs. (3.12)-(3.17). In the warped case, the contribution from massive KK states to the running of operators on the boundaries $z = z_0, z_1$ will freeze out at the typical local scale $1/z_0$, $1/z_1$. This gives the $\log \mu z_0$, $\log \mu z_1$ terms in $\Pi(p^2,\mu)$ which are the counterpart of the $\log \mu R$ terms of the flat case. A further source of $\log z_{0,1}$ terms might be finite non-local operators which will be in general present in the 5d effective action. Indeed, the dependence on $z_{0,1}$ of the function f in eq. (3.11), is quite complicated before taking the limit $z_0 \ll z_1$. Only when the is a large separation of scales $z_0 \ll z_1$, we recover the simple expression of eqs. (3.12)-(3.15) required by the holographic interpretation.

Concerning the holographic interpretation of the brane kinetic terms in AdS, they correspond to adding a constant term to the 4d inverse coupling $1/g^2(p^2)$, shifting its Landau pole [39]. In other words, it is a modification of the 4d theory at a scale corresponding to the position of the brane in AdS. There is therefore no connection between boundary terms in AdS and log evolution in the holographic theory. It is remarkable that in the flat case all the logarithmic running comes from boundary operators, while the main logarithmic running in the 4d theory dual to RSI comes from the AdS bulk.

3.3 GUT breaking: the holographic point of view

Armed with the previous results, we discuss now different mechanisms of breaking the GUT symmetry in AdS, either through suitable boundary conditions for the gauge fields, or turning on the vev of a scalar field in the bulk. We consider for simplicity the particular case of an $SU(5)$ group in the bulk broken down to $SU(3)\times SU(2)\times U(1)$ and we study the loop correction to the low energy couplings given by a scalar multiplet in the fundamental representation. It is understood that the results have a general validity.

3.3.1 GUT breaking through boundary conditions

Let us consider first the case in which the GUT symmetry is reduced at low energy by the boundary conditions. We assume that the $SU(3)\times SU(2)\times U(1)$ gauge bosons A^a_μ have always parity $(++)$, while the X,Y bosons $A^{\hat{a}}_\mu$ can be $(\pm,\mp)$ or $(--)$: $SU(5)$ is broken on the TeV or Planck brane, or both. The relative parities of the doublet and triplet components of the scalar 5-plet φ in the bulk are fixed by gauge invariance. We choose $\varphi_2=(++)$ for the doublet component and this forces $\varphi_3=(\pm,\mp)$, $(--)$ for the triplet when $A^{\hat{a}}_\mu$ are $(\pm,\mp)$, $(--)$ respectively.

GUT BREAKING ON THE TEV BRANE

$$\varphi=\begin{bmatrix}\varphi_2(++)\\ \varphi_3(+-)\end{bmatrix}\qquad\qquad \text{for}\quad A^{\hat{a}}_\mu(+-)\ A^{\hat{a}}_5(-+)$$

A theory with a gauge group $SU(5)$ in pure AdS is dual to a 4d CFT with a *global* $SU(5)$ invariance. Putting the Planck brane and imposing $+$ conditions for the gauge bosons corresponds, in the holographic theory, to gauge the global symmetry. Let us now insert the TeV brane demanding $-$ parity for the X,Y (and $+$ for the A^a_μ) gauge fields. This deformation in AdS implies in the 4d picture a spontaneous breaking of $SU(5)$ down to the $SU(3)\times SU(2)\times U(1)$ subgroup at the TeV: the X,Y bosons acquire TeV masses through the Higgs mechanism and the CFT resonances are not $SU(5)$ invariant. At energies higher than the TeV, however, the Planck brane-brane correlator does not probe the GUT breaking on the TeV brane and the holographic theory must appear fully $SU(5)$ invariant. As a consequence, we expect a GUT-invariant running of the $SU(3)\times SU(2)\times U(1)$ gauge couplings g_i, $i=1,2,3$, from the TeV up to higher energies. This is indeed what we found computing the contribution of the massless 5-plet scalar (for $1/z_1\gg|p|\gg z_0/z_1^2$):

$$\begin{aligned}\frac{1}{g_i^2(p^2)}=&\frac{1}{8\pi^2}\log\frac{z_1}{z_0}\left[\frac{8\pi^2}{kg_5^2}-b_5\right]+\Delta_0(1/z_0)+\Delta_1^i(1/z_1)-\frac{b_5}{8\pi^2}\log z_0\sqrt{-p^2}\\ &-\frac{1}{8\pi^2}\left[\frac{b_2^i}{2}\left(-\frac{1}{\varepsilon}+\frac{\gamma}{2}-\frac{8}{3}\right)-\frac{b_3^i}{2}\left(2\log 2+\frac{8}{3}\right)\right].\end{aligned}\tag{3.18}$$

We denote with $b^i_{2,3}$ the beta-functions of a 4d scalar doublet, triplet respectively and with b_5 the $SU(5)$-invariant beta-function. We recognize in the previous formula (fourth term) the contribution of the holographic 5-plet (a massless doublet and a triplet with a tiny mass $\sim \mathrm{TeV}^2/k$), and the CFT contribution at NLO in $1/N$ (first term). Both are $SU(5)$ invariant as expected [8]. Notice that for this to happen, we had to evaluate the boundary couplings $\Delta_{0,1}(\mu)$ on the Planck and TeV branes at $\mu = k, \mathrm{TeV}$ respectively. This is quite natural as these boundary terms $\Delta_{0,1}$ correspond holographically to threshold corrections at the scales $1/z_0$, $1/z_1$. Had we evaluated, for instance, the TeV boundary term at $\mu = k$, a fake $SU(5)$-breaking effect would have been introduced, coming from the $SU(5)$ non-invariant evolution of $\Delta^i_1(\mu)$. The logarithmic divergence is canceled with an $SU(5)$ non-invariant counterterm on the TeV brane: the only source of differentiation among the three couplings g_i comes from $\Delta^i_1(1/z_1)$ and from some finite scheme-dependent terms absorbable in Δ^i_1. Changing the value of the latter, corresponds holographically to modify the Higgs mechanism responsible for the $SU(5)$ breaking. How much the g_i depart from a common value below the TeV depends therefore on the unknown value of $\Delta^i_1(1/z_1)$. It is clearly an important phenomenological question to estimate this contribution in some way. One can advocate a plausible strong coupling hypothesis [71] assuming that the $\Delta^i_1(\mu)$ are sufficiently small when the gauge dynamics becomes strongly coupled. On the TeV brane this happens at energies $\mu \gtrsim \mathrm{TeV}$, confirming that the choice of the scale $\mu = 1/z_1$ for Δ_1 is the correct one.

GUT BREAKING ON BOTH THE TEV AND PLANCK BRANE

$$\varphi = \begin{bmatrix} \varphi_2(++) \\ \varphi_3(--) \end{bmatrix} \qquad\qquad \text{for} \quad A^{\hat a}_\mu(--) \;\; A^{\hat a}_5(++)$$

If $SU(5)$ is broken by the Planck brane boundary conditions, the holographic theory does not have X,Y bosons. Even if the CFT has a global $SU(5)$ invariance (see section 3.1), only the $SU(3)\times SU(2)\times U(1)$ symmetry is gauged. In the holographic theory we thus find, in addition to the CFT sector, the $SU(3)\times SU(2)\times U(1)$ gauge fields and an elementary doublet scalar. Inserting the TeV brane in AdS and demanding a $-$ parity for the $A^{\hat a}_\mu$, the global $SU(5)$ invariance of the CFT is spontaneously broken at the TeV to the $SU(3)\times SU(2)\times U(1)$ subgroup. The corresponding Goldstone bosons can be identified with the zero modes of $A^{\hat a}_5$, which appear in the dual theory as scalar excitations of the CFT with the same quantum numbers of the XY bosons. From the holographic point of view, we thus expect an $SU(5)$-breaking running up to the Planck scale given by the scalar doublet, while the CFT does not contribute to the differential running. Indeed the

[8]At very low energies, $|p| < z_0/z_1^2$, the triplet contribution stops and the running becomes different for the three $SU(3)\times SU(2)\times U(1)$ couplings g_i.

explicit calculation gives:

$$\begin{aligned}\frac{1}{g_i^2(p^2)} =& \frac{1}{8\pi^2}\log\frac{z_1}{z_0}\left[\frac{8\pi^2}{kg_5^2} - b_5\right] + \Delta_0^i(1/z_0) + \Delta_1^i(1/z_1) - \frac{b_2^i}{8\pi^2}\log z_0\sqrt{-p^2} \\ & - \frac{1}{8\pi^2}\left[\frac{(b_2^i - b_3^i)}{2}\left(-\frac{1}{\varepsilon} + \frac{\gamma}{2}\right) - \frac{b_3^i}{2}\log 2 - b_2^i\frac{4}{3}\right] .\end{aligned} \tag{3.19}$$

This equation gives a non-ambiguous test of the holographic interpretation: the 4d scalar doublet gives a differential running up to the scale k. This effect cannot be falsified by the $SU(5)$-invariant running of the CFT.

An important observation is in order at this point. From the holographic point of view, there is no reason at all why the different g_i couplings should unify at the Planck scale. Indeed, in the holographic theory $SU(5)$ is just a global symmetry of the pure CFT sector, only the $SU(3)\times SU(2)\times U(1)$ group is gauged. This is in sharp contrast with the case of $SU(5)$ broken only by the TeV brane: in that case, there is a Higgs mechanism in 4d reducing the GUT group at the TeV. No analogous mechanism arises here at the Planck scale. Moreover, from the 5d point of view, the situation at energies around k is similar to the flat case: there is really no exact unification of the gauge couplings just because there is no unified symmetry on the boundaries. As in the flat limit, however, one can estimate the threshold corrections, represented in AdS by the boundary term $\Delta_0^i(\mu)$, to be small if evaluated at a scale $\mu \sim 1/z_0$ close to the strong dynamics regime. In this sense, we recover an approximate unification of the couplings g_i at the Planck scale.

GUT BREAKING ON THE PLANCK BRANE

$$\varphi = \begin{bmatrix}\varphi_2(++)\\ \varphi_3(-+)\end{bmatrix} \qquad\qquad \text{for} \quad A_\mu^{\hat a}(-+)\ A_5^{\hat a}(+-)$$

The GUT symmetry is still broken on the Planck brane but no more on the TeV, so that the holographic picture is much similar to the previous case. Inserting a TeV brane and demanding a $+$ parity for the $A_\mu^{\hat a}$, it means that $SU(5)$ remains a global symmetry of the CFT: the CFT resonances can be arranged in exact $SU(5)$ multiplets. As in the previous case we expect that the only source of $SU(5)$ breaking comes from the scalar doublet. Indeed we obtain

$$\begin{aligned}\frac{1}{g_i^2(p^2)} =& \frac{1}{8\pi^2}\log\frac{z_1}{z_0}\left[\frac{8\pi^2}{kg_5^2} - b_5\right] + \Delta_0^i(1/z_0) + \Delta_1(1/z_1) - \frac{b_2^i}{8\pi^2}\log z_0\sqrt{-p^2} \\ & - \frac{1}{8\pi^2}\left[\frac{b_2^i}{2}\left(-\frac{1}{\varepsilon} + \frac{\gamma}{2} - \frac{8}{3}\right) + \frac{b_3^i}{2}\log 2\right] .\end{aligned} \tag{3.20}$$

3.3.2 GUT breaking with a bulk vev

A different mechanism to break the GUT symmetry is the standard Higgs mechanism. Let us suppose that a massless scalar field Σ, propagating in the bulk, acquires a vacuum expectation value $\langle\Sigma\rangle$ constant along the fifth dimension. In the following we assume that Σ and all the other bulk fields have $(++)$ boundary conditions. This vev splits the masses of the GUT multiplets, giving, for example, a (bulk) mass $m \sim g_5\langle\Sigma\rangle$ to the triplet of our scalar φ, leaving the doublet massless. An interesting possibility is that $k \gg m \gg$ TeV so that the one-loop correction to the low energy couplings reads:

$$\begin{aligned}\frac{1}{g_i^2(p^2)} =& \frac{1}{8\pi^2}\log\frac{z_1}{z_0}\left[\frac{8\pi^2}{kg_5^2} - b_5 - b_3^i\frac{m^2z_0^2}{8}\right] \\ &+\Delta_0(1/z_0)+\Delta_1(1/z_1) - \frac{b_2^i}{8\pi^2}\log\frac{\sqrt{-p^2}}{m} - \frac{b_5}{8\pi^2}\log mz_0 \\ &-\frac{1}{8\pi^2}\left[\frac{b_5}{2}\left(-\frac{1}{\varepsilon}+\frac{\gamma}{2}\right) - \frac{4}{3}b_2^i - \frac{b_3^i}{2}\log 2 - b_3^i\frac{m^2z_0^2}{16}\right] .\end{aligned} \tag{3.21}$$

In the 4d dual picture the gauge symmetry is spontaneously broken. The 4d doublet and triplet scalars take different masses (the triplet has a mass $\sim m/\sqrt{2}$, corresponding to the lowest eigenvalue of the 5d KK tower): their contribution can be recognized in eq. (3.21). The vev of the Σ field implies, as discussed in section 3.1, that the CFT is not $SU(5)$ invariant. Therefore, we expect GUT symmetry breaking terms in the CFT beta-function proportional to m^2/k^2; in fact they appear in the first term of eq. (3.21). While the $\log m$ can be traced back to a calculable and non-analytic operator $(\log\Sigma)FF$ in the AdS effective action, the m^2/k^2 terms come from Σ^2FF operators on the AdS side. As a last remark, we notice that there are no terms in eq. (3.21) linear in m. They would be the counterpart either of an analytic 5d operator ΣFF or of the non-analytic operator $\sqrt{\Sigma^2}FF$. The first one is absent if we impose a $\Sigma \to -\Sigma$ symmetry and the second one shows up only in the flat limit $m \sim g_5\langle\Sigma\rangle \gg k$, as already said in section 3.2.

Some conclusions on different GUT scenarios can be drawn. First we try to answer the question posed in the introduction, if we can use the Randall-Sundrum model to both solve the hierarchy problem and address gauge couplings unification *without supersymmetry*. In order to satisfy the first request, we need to put at least the Higgs on the TeV brane. Then we must break the GUT gauge symmetry. We know that in the 4d non-supersymmetric SU(5) model, the gauge couplings come close at high energy, but they do not unify. An exact unification would require an extra contribution to the differential running, which is $\sim 20\%$ of that coming from the Standard Model fields. We can imagine that in our model the CFT will help, but the leading CFT contribution is common to all gauge groups and the subleading correction will contribute to the differential running only if the SU(5) symmetry is broken in the bulk by the Higgs mechanism. In the 4d standard scenario, Planck suppressed thershold corrections, coming from non-renormalizable operators involving the Higgs field like $\Sigma FF/M_4$, cannot account for such an effect. In the 5d AdS theory, on the other hand, these threshold receive a volume factor enhancement $\log(z_1/z_0)$. In the holographic theory, the contribution of the bulk operator $\Sigma FF/\sqrt{\Lambda}$ is the correction to the CFT beta-function, so that the log enhancement is explained as the effect of the running up to high energies. In [74] it was observed that, considering these thresholds, gauge couplings unification can be achived with sufficient precision in the case of a Randall-Sundrum SU(5) GUT model, even wihout supersymmetry. We see from our NDA estimate (3.5) that for $M_{GUT}/\Lambda \sim 0.2$, the CFT correction is of the correct order of magnitude. We stress that this unification relies on the uncalculable effect of non-renormalizable local operators.

Other scenarios could be interesting for model building. The GUT group can be broken on the boundaries and not in the bulk; in this case the uncalculable CFT contribution is exactly GUT-invariant at energies greater than the TeV, and the whole differential running comes from the elementary modes. If the Standard Model particles are confined on the Planck brane, supersymmetry is required to stabilize the hierarchy; one then reobtains a standard supersymmetric unification, if a spontaneous breaking occurs on the Planck brane [29]. From the 4d point of view, we have just added to the MSSM a GUT-invariant CFT, which gives a common positive contribution to all the three beta-functions. However the phenomenology of this scenario drastically differs from that of the MSSM, because the CFT bound states are produced at TeV in multiplets of the GUT group.

If the breaking on the Planck brane proceeds through boundary conditions, there is no unification in the usual sense, as only the SM gauge bosons exist in the holographic dual . Nevertheless, as in the flat case, an approximate unification at high energies can be justified from a 5d point of view, relying on a strong coupling assumption for the boundary couplings on the Planck brane. From the holographic point of view, the CFT has a global invariance under the GUT symmetry, but only the SM subgroup is gauged. The unification is driven by the strong dynamics of the CFT: the positive and GUT-symmetric contribution to the CFT beta function, makes the couplings to grow with energy until they become strong and are forced to be equal. A realistic model of this kind has been constructed in [75].

Another possibility is that the GUT symmetry is broken through TeV brane boundary conditions. This breaking is negligible for energies above the TeV scale; to address unification, additional sources of GUT breaking are therefore required, such as a Higgs mechanism in the bulk or on the Planck brane. If the only source of symmetry breaking is the choice of boundary conditions on the TeV brane, the unification scale should be at the TeV scale. This could fit well in the framework of $SU(3)_W$ unification [76], recently readdressed in extra-dimensional inspired models [77], in which the $SU(2)$ and $U(1)$ groups of the Standard Model are embedded into a weak $SU(3)$ around the TeV scale. However, it is likely that this kind of model requires a scale of conformal symmetry breaking too low to be compatible with the strong limits coming from electroweak precision observables [78].

Appendix A

Component Lagrangian

In this appendix, we describe the component form of the superfield Lagrangian (2.69). Apart from the terms involving the prepotential $P_{\mathscr{T}}$ for the radion, the calculation is very similar to the one done in ref. [54], and we therefore refer the reader to that paper for more details. The first step is to chose a convenient Wess–Zumino gauge as in [54], in which the bosonic components of the superfields are given by the following expressions:

$$\begin{aligned}
V_n &= -\theta\sigma^m\bar{\theta}\tilde{h}_{mn} + \theta^2\bar{\theta}^2 d_n \ , \\
\Psi_\alpha &= \theta^\beta u_{\beta\alpha} + \frac{i}{2}\bar{\theta}_{\dot{\alpha}}v_\alpha{}^{\dot{\alpha}} + \bar{\theta}^2\theta^\beta\left(w_{\beta\alpha} + \frac{1}{4}\sigma^n_{\beta\dot{\beta}}\partial_n v_\alpha{}^{\dot{\beta}}\right) + \theta^2\bar{\theta}_{\dot{\alpha}}\left(y_\alpha{}^{\dot{\alpha}} + \frac{i}{2}\bar{\sigma}^{n\dot{\alpha}\beta}\partial_n u_{\beta\alpha}\right) , \\
P_\Sigma &= -\theta\sigma^m\bar{\theta}\tau^\Sigma_m + \theta^2\bar{\theta}^2 D_\Sigma \ , \\
\Sigma &= \theta^2\left(D_\Sigma - \frac{i}{2}\partial_m\tau^\Sigma_m\right) , \\
P_{\mathscr{T}} &= \rho_{\mathscr{T}} + \theta^2\bar{t} + \bar{\theta}^2 t - \theta\sigma^m\bar{\theta}\tau^{\mathscr{T}}_m + \theta^2\bar{\theta}^2\left(D_{\mathscr{T}} - \frac{1}{4}\Box\rho_{\mathscr{T}}\right) , \\
\mathscr{T} &= t + \theta^2\left(D_{\mathscr{T}} - \frac{i}{2}\partial_m\tau^{\mathscr{T}}_m\right) + i\theta\sigma^m\bar{\theta}\partial_m t + \frac{1}{4}\theta^2\bar{\theta}^2\Box t \ .
\end{aligned}$$

In components this give:

$$\begin{aligned}
\mathscr{L} = e^{-2\sigma}\Bigg\{ & \frac{1}{2}(\partial^n\tilde{h}_{nm})^2 + \frac{1}{2}(\partial^n\tilde{h}_{mn})^2 - \frac{1}{2}(\partial_p\tilde{h}_{mn})^2 + \frac{1}{3}\tilde{h}\partial_m\partial_n\tilde{h}_{mn} \\
& + \frac{1}{6}(\partial_m\tilde{h})^2 - \frac{1}{6}(\varepsilon^{mnpq}\partial_p\tilde{h}_{mn})^2 + \frac{4}{3}d_n^2 + \frac{2}{3}\varepsilon^{mnpq}\partial_p\tilde{h}_{mn}d_q \\
& + \frac{1}{4}\left(\sigma^n_{\beta\dot{\alpha}}w^\beta{}_\alpha + \sigma_{\alpha\dot{\beta}}\,\bar{w}^{\dot{\beta}}{}_{\dot{\alpha}} + \mathrm{Re}\partial_n v_{\alpha\dot{\alpha}} - e^{-\sigma}\partial_y\tilde{h}_{n\alpha\dot{\alpha}}\right)^2 \\
& + \frac{1}{2}\mathrm{Im}v_n\Box\mathrm{Im}v^n + 2e^{-\sigma}\mathrm{Im}v_n\partial_y d^n + \frac{i}{2}\mathrm{Im}v^{\alpha\dot{\alpha}}\partial_{\beta\dot{\alpha}}w^\beta{}_\alpha - \frac{i}{2}\mathrm{Im}v^{\dot{\alpha}\alpha}\partial_{\alpha\dot{\beta}}\,\bar{w}^{\dot{\beta}}{}_{\dot{\alpha}} \\
& + \frac{1}{2}\left|w^\alpha{}_\alpha + \partial_n v_n - 3e^{-\sigma}\sigma' t\right|^2 + \mathrm{Re}t\,\partial_m\partial_n\tilde{h}_{mn} + 2\left|y_n\right|^2 \\
& - 2\left[\mathrm{Re}y_n + \frac{i}{4}\bar{\sigma}^{n\dot{\alpha}\beta}\left(\partial_{\alpha\dot{\alpha}}u^\alpha{}_\beta - \partial_{\beta\dot{\beta}}\,\bar{u}^{\dot{\beta}}{}_{\dot{\alpha}}\right) - \frac{1}{4}e^{-\sigma}\left(\partial_y\tau^\Sigma_m + 3\sigma'\tau^{\mathscr{T}}_m\right)\right]^2 \\
& - \frac{3}{2}e^{-\sigma}\sigma'\rho_{\mathscr{T}}\left[2\partial_p\mathrm{Im}y_p - e^{-\sigma}\left(\partial_y D_\Sigma + 3\sigma'(D_{\mathscr{T}} - \frac{1}{4}\Box\rho_{\mathscr{T}})\right)\right]
\end{aligned}$$

$$-D_{\mathscr{T}} D_\Sigma - \frac{1}{4}\partial_m \tau_m^{\mathscr{T}} \partial_m \tau_m^\Sigma - \frac{1}{3} D_\Sigma^2 - \frac{1}{12}(\partial_m \tau_m^\Sigma)^2 \Big\} \, . \tag{A.1}$$

There are two independent sectors: one containing h, t, v, w and d, and one containing $D_\Sigma, D_{\mathscr{T}}, \tau_m^\Sigma, \tau_m^{\mathscr{T}}, y_n, u_{\alpha\beta}$. In the former, h, t and v are physical fields, whereas w and d are auxiliary that must be integrated out. In the latter, all of the fields are non-propagating fields. After eliminating all the auxiliary and non-propagating fields, one finally finds the following Lagrangian:

$$\begin{aligned}
\mathscr{L} = M_5^3 e^{-2\sigma} \Big\{ & (\partial^n h_{nm})^2 + h\partial_n\partial_m h_{nm} + \frac{1}{2}(\partial_m h)^2 - \frac{1}{2}(\partial_p h_{mn})^2 \\
& + \frac{1}{2} e^{-2\sigma}\left[(\partial_y h)^2 - (\partial_y h_{mn})^2\right] - \frac{1}{2}\left(\partial_n h_{my} - \partial_m h_{ny}\right)^2 \\
& + 2e^{-\sigma}\partial_m h_{ny}\partial_y h^{mn} - 2e^{-\sigma}\partial^n h_{ny}\partial_y h + h_{yy}\partial^m\partial^n h_{mn} - h_{yy}\Box h \\
& + 6e^{-2\sigma}\sigma'^2 h_{yy}^2 - 6e^{-\sigma} h_{yy}\partial_n h_{ny} + 3e^{-2\sigma}\sigma' h_{yy}\partial_y h \\
& - \frac{1}{4} e^{2\sigma}\left(\partial_n B_m - \partial_m B_n\right)^2 - \frac{1}{2}\left(\partial_m B_y - \partial_y B_m\right)^2 \Big\} \, ,
\end{aligned} \tag{A.2}$$

where we have defined:

$$h_{mn} = \frac{1}{2}(\tilde{h}_{mn} + \tilde{h}_{nm}) - \frac{1}{3}\eta_{mn}\tilde{h} \, , \;\; h_{my} = \frac{1}{2}\mathrm{Re}\, v_m \, , \;\; h_{yy} = \mathrm{Re}\, t \, ; \tag{A.3}$$

$$B_m = e^{-\sigma}\sqrt{\frac{3}{2}}\mathrm{Im}\, v_m \, , \;\; B_y = \sqrt{\frac{3}{2}}\mathrm{Im}\, t \, . \tag{A.4}$$

This Lagrangian match by expanding the full RS1 Lagrangian to quadratic order in fluctuations parameterized in the following way:

$$ds^2 = e^{-2\sigma}\left(\eta_{mn} + h_{mn}\right) dx^m dx^n + 2e^{-\sigma} h_{my} dx^m dy + (1 + h_{55}) dy^2 \tag{A.5}$$

In fact, a better parametrization of the fluctuation, that makes the zero mode manifest is:

$$ds^2 = e^{-2\sigma(1+T)}\left(\eta_{mn} + h_{mn}\right) dx^m dx^n + 2e^{-\sigma} h_{my} dx^m dy + (1+T)^2 dy^2 \tag{A.6}$$

It can be checked that $T = \overline{T}(x)$ and $h_{mn} = \overline{h}_{mn}(x)$ are zero modes. This fact can be seen in our quadratic Lagrangian by replacing h_{mn} by $\overline{h}_{mn}(x) - \sigma\overline{h}_{yy}(x)$, h_{yy} by $\overline{h}_{yy}(x)$, B_y by $\overline{B}_y(x)$,setting the odd field h_{my} and B_m to zero, and integrating over y. We get:

$$\begin{aligned}
\int d^4x M_5^3 \Big\{ & \left(\frac{1 - e^{-2k\pi R}}{k}\right)\Big((\partial^n h_{nm})^2 + h\partial^m\partial^n h_{mn} + \frac{1}{2}(\partial_m h)^2 - \frac{1}{2}(\partial_p h_{mn})^2 \\
& - \frac{1}{2}(\partial_m B_y)^2 \Big) + e^{-2k\pi R}\left(\pi R\left(\partial^n\partial^m h_{mn} - \Box h\right) h_{yy} + 3k\pi^2 R^2 h_{yy}\Box h_{yy}\right) \Big\}
\end{aligned} \tag{A.7}$$

Appendix B

Sums in AdS

We present here the method used to sum the series of eq. (3.10) on the AdS Kaluza-Klein masses. Performing first the integral in eq. (3.10), one find the series

$$S(d) = \sum_{\{x_n\}} \left(x_n^2 + c^2(x)\right)^{d/2-2} \ , \tag{B.1}$$

where the summation runs over the entire KK spectrum of the scalar field. Depending on its boundary conditions, the KK masses x_n of a massive scalar field in AdS satisfy the following eigenvalue equations:

$$\begin{aligned} (++):& \quad \frac{j_\nu(x_n z_0)}{y_\nu(x_n z_0)} = \frac{j_\nu(x_n z_1)}{y_\nu(x_n z_1)}; \qquad & (--):& \quad \frac{J_\nu(x_n z_0)}{Y_\nu(x_n z_0)} = \frac{J_\nu(x_n z_1)}{Y_\nu(x_n z_1)} \\ (+-):& \quad \frac{j_\nu(x_n z_0)}{y_\nu(x_n z_0)} = \frac{J_\nu(x_n z_1)}{Y_\nu(x_n z_1)}; \qquad & (-+):& \quad \frac{J_\nu(x_n z_0)}{Y_\nu(x_n z_0)} = \frac{j_\nu(x_n z_1)}{y_\nu(x_n z_1)} \end{aligned} \tag{B.2}$$

Here J_ν, Y_ν are Bessel functions, $\nu = \sqrt{4 + m^2 z_0^2}$, with m the 5d mass, and

$$y_\nu(z) = Y_{\nu-1}(z) + \frac{(2-\nu)}{z} Y_\nu(z); \qquad j_\nu(z) = J_{\nu-1}(z) + \frac{(2-\nu)}{z} J_\nu(z) \ . \tag{B.3}$$

Choosing the functions:

$$\begin{aligned} f_{++}(z) &= y_\nu(zz_1)\, j_\nu(zz_0) - y_\nu(zz_0)\, j_\nu(zz_1) \\ f_{--}(z) &= Y_\nu(zz_1) J_\nu(zz_0) - Y_\nu(zz_0) J_\nu(zz_1) \\ f_{+-}(z) &= i\left[y_\nu(zz_0) J_\nu(zz_1) - Y_\nu(zz_1)\, j_\nu(zz_0)\right] \\ f_{-+}(z) &= i\left[y_\nu(zz_1) J_\nu(zz_0) - Y_\nu(zz_0)\, j_\nu(zz_1)\right] \ , \end{aligned} \tag{B.4}$$

whose zeros are the x_ns, one can rewrite the sum in eq. (B.1) as a complex integral over the contour Γ with $R \to \infty$ (see fig. B.1):

$$S(d) = \frac{1}{2\pi i} \int_\Gamma dz \, \left(z^2 + c^2\right)^{d/2-2} \, \frac{f'(z)}{f(z)} \tag{B.5}$$

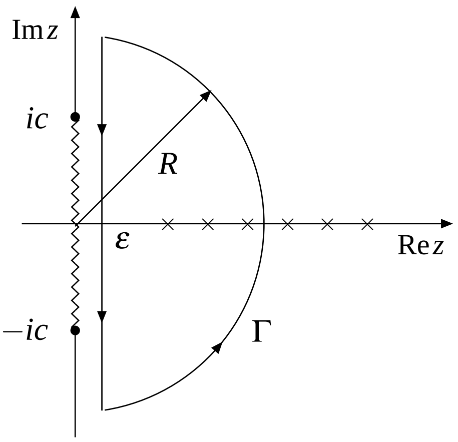


Figure B.1: Contour Γ in the complex plane. The crosses along the real axis correspond to the real positive zeros x_n of the function f.

with f one of the functions in eq. (B.4) [1]. What follows applies for a generic parity, and therefore we will specify the function f only when necessary. The asymptotic expansion of $f(z)$ when $\mathrm{Im}\, z \to \pm\infty$, the same for all the parities,

$$\frac{f'(z)}{f(z)} = -\left(\pm i(z_1 - z_0) + \frac{1}{z}\right) + O(1/z^2) \tag{B.6}$$

tells us that the integral, like the original series, converges at infinity ($R \to \infty$) if $d < 3$. In order to find the expression of $S(d)$ for $d \to 4$, we first take $d < 3$ and extract the limit $R \to \infty$. The contribution of the integration around the semi-circle of radius R goes to zero and we are left with the vertical contour. Let us call for convenience Γ^+, Γ^- the part of this vertical contour respectively above, below the real axis. We now subtract the asymptotic behaviour of f'/f and evaluate it separately deforming Γ^+ and Γ^- to coincide with the real axis. Defining

$$F(z) = \frac{f'(z)}{f(z)} + \frac{1}{z} + i(z_1 - z_0) \tag{B.7}$$

and using the parity properties $f_{\pm\pm}(-z) = f_{\pm\pm}(z)$, $f_{\pm\mp}(-z) = -f_{\pm\mp}(z)$, we obtain:

$$\begin{aligned} S(d) =& \frac{1}{2\pi i}\left[\int_{\Gamma^+} dz\, \left(z^2 + c^2\right)^{d/2-2} F(z) - \int_{\Gamma^-} dz\, \left(z^2 + c^2\right)^{d/2-2} F(-z)\right] \\ &+ \frac{(z_1 - z_0)}{2\sqrt{\pi}} \left(c^2\right)^{(d-3)/2} \frac{\Gamma\left(\frac{3-d}{2}\right)}{\Gamma(2-d/2)}\,. \end{aligned} \tag{B.8}$$

In the remaining integrals, we can now extract the limit $d \to 4$, being $F(z) \sim 1/z^2$ at infinity. We expand the integrand up to orders $O[(d-4)^2]$. The first term in the expansion

[1] Even if the domain of definition of the Bessel functions $J_\nu(z)$, $Y_\nu(z)$ is the z-plane cut along the negative real axis, the functions $f_{\pm,\pm}(z)$, $f_{\pm\mp}(z)$ are single-valued on the entire complex plane.

gives a non-vanishing result because of a residue contribution in the origin (here we deform the contours Γ^+, Γ^- to coincide with the imaginary axis, $\varepsilon \to 0$ in fig. B.1):

$$\frac{1}{2\pi i}\left[\int_{\Gamma^+} dz F(z) - \int_{\Gamma^-} dz F(-z)\right] = \frac{\alpha}{2}; \qquad \alpha = \begin{cases} \pm 1 & \text{for } (\pm,\pm) \\ 0 & \text{for } (\pm,\mp) \end{cases} . \tag{B.9}$$

The second term in the expansion must be evaluated taking into account the cut along the imaginary axis between $\pm ic$. We find:

$$\begin{aligned} &\frac{1}{2\pi i}\left[\int_{\Gamma^+} dz \log\left(z^2+c^2\right) F(z) - \int_{\Gamma^-} dz \log\left(z^2+c^2\right) F(-z)\right] \\ &\qquad = \log f(ic) + \log c\pi\sqrt{z_0 z_1} + \alpha \log c - c(z_1 - z_0) . \end{aligned} \tag{B.10}$$

Summing all the contributions we get our final result

$$\sum_{\{x_n\}} \left(x_n^2 + c^2(x)\right)^{d/2-2} = \frac{\alpha}{2} + (d/2-2)\left[\log f(ic) + \log c\pi\sqrt{z_0 z_1} + \alpha \log c\right] + O[(d-4)^2] \tag{B.11}$$

which leads to eq. (3.11). Finally, we write the $z \to 0$ expansion of the various functions f, used in the text to obtain the low energy limit of $\Pi(p^2,\mu)$. Taking only the relevant terms, one has ($z \to 0$, $z_1 \gg z_0$):

$$\begin{aligned} f_{++}(z) &\simeq \frac{1}{\pi\nu}\left(\frac{z_1}{z_0}\right)^{\nu-1}\left[\frac{4-\nu^2}{z^2 z_0^2} + \frac{2+\nu}{2(\nu-1)}\right] \\ f_{--}(z) &\simeq \frac{1}{\pi\nu}\left(\frac{z_1}{z_0}\right)^{\nu} \\ f_{+-}(z) &\simeq \frac{i}{\pi\nu}\left(\frac{z_1}{z_0}\right)^{\nu}\left[\frac{\nu-2}{zz_0} - \frac{zz_0}{2(\nu-1)} + \frac{\nu+2}{zz_0}\left(\frac{z_0}{z_1}\right)^{2\nu}\right] \\ f_{-+}(z) &\simeq \frac{i}{\pi\nu}\left(\frac{z_1}{z_0}\right)^{\nu}\frac{2+\nu}{zz_1} . \end{aligned} \tag{B.12}$$

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